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Effect of Inclination Angle of The Side Walls on The Natural Convection Heat Transfer Inside an Enclosure

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Abstract

Laminar natural convection heat transfer and fluid flow due to the heating from below at variable heat source length inside two dimensional enclosure has been analyzed numerically in this study. The enclosure has filled with air as a working fluid. The vertical inclined walls of the enclosure are maintained at lower temperature while the remaining walls are insulated. The value of Rayleigh number from ($1x10^3 \leq Ra \leq 4x10^4$), the inclination angle at ($\gamma = 0^o$, 22.5^o , 45^o) and dimensionless heat source length at ($S = 1$ and 0.5). The continuity, momentum and energy equations have been applied to the enclosure and solved by using finite difference method. The results showing that the average Nusselt number increases with the increasing of the heating source length and decreases with the increasing in an inclination angle of the vertical walls.

keywords: **Heat transfer , Natural convection, Enclosure, Heated from below, Inclination of vertical walls.**

1. Introduction:

The phenomenon of natural convection heat transfer and fluid flow or some times called buoyancy-driven flow in enclosures has been studied extensively in the recent years in response to energy-related applications, such as thermal insulation of buildings by using air gaps, solar energy collectors, furnaces and fire control in buildings. Most of the geometrical configurations of the enclosures in the previous studies are focused on rectangular or square enclosure. However, the shape of the enclosure can be in different configurations such as, in most of the related engineering situations which include triangle, parallelogram or trapezoidal. [1]

Among the earlier reported studies for rectangular, trapezoidal and triangular enclosures. M. Moghimi and et al [2] investigated the natural convection in 2-D rectangular enclosures filled with air, heated from below and cooled from above numerically. For a wide variety of thermal boundary conditions at the sidewalls. Simulations are performed for several values of both the width-to-height aspect ratio of the enclosure in the range between 0.25 and 1, and the Rayleigh number based on the cavity height in the range between 1.00e3 and 5.00e5, whose influence upon the flow patterns, the temperature distributions and the heat transfer rates are analyzed and discussed.

E. Natarajan and et al [3] used a penalty finite element analysis with bi-quadratic elements is performed to investigate the influence of uniform and non-uniform heating of bottom wall on natural convection flows in a trapezoidal cavity. The bottom wall is uniformly and non-uniformly heated while the two vertical walls are maintained at constant cold temperature and the top wall is well insulated. Parametric study for the wide range of Rayleigh number (Ra), $10^3 \leq$ Ra $\leq 10^5$ and Prandtl number (Pr), $0.07 \leq$ Pr ≤ 100. Results represented in terms of stream functions, temperature profiles and the Nusselt number. They found the average Nusselt number shows overall lower heat transfer rate for non-uniform heating case and the effect of Prandtl number in the variation of local and average Nusselt numbers is more significant for Prandtl numbers in the range 0.07–0.7 than 10–100.

Ahmed W. Mustafa and et al [4] investigated the natural convection in a trapezoidal enclosure with partial heating from below and symmetrical cooling from the sides. The heating is simulated by a centrally located heat source on the bottom wall, and four different values of the dimensionless heat source length, 1/5, 2/5, 3/5, 4/5 are considered. The range of Rayleigh number is ($10^3 \leq$ Ra $\leq 10^5$) and Prandtl number is 0.7. They found that the average Nusselt number increases with the increase of the source length.

Goutam Sahar and et al [5] studied the natural convection in tilted isosceles triangular enclosure filled with air. Two upper walls are maintained at constant cold temperature, whereas a constant heat flux is symmetrically embedded at the bottom wall, and the non-heated parts of the bottom-wall are considered adiabatic. This study reports the effect of various aspect ratios, ranging from 0.5 to 1, and inclination angles of the enclosure from 0^o to 60^o. Results are presented in the form of streamline and isotherm plots as well as the variations of the Nusselt number and maximum temperature at the heat source surface under different conditions.

Orhan Aydin and et al [6] investigated numerically the natural convection of air in a two-dimensional rectangular enclosure with localized heating from below and symmetric cooling from the sides. Localized heating is simulated by a centrally located heat source on the bottom wall, the solution obtained for different values of dimensionless heat source length, 1/5 , 2/5 , 3/5 , 4/5 and different Rayleigh number values from 10^3 to 10^6. The results shows that the average Nusselt number at heated part of the lower wall, $\overline{Nu}$, increase with an increase the Rayleigh number, or of the non-dimensional heat source thickness.

The main purpose of this study is to investigate the laminar natural convection heat transfer and fluid flow due to insert the effect of heat source length and inclination angle of the vertical walls of the enclosure. The literature review given above shows that, natural convection in square, rectangle, triangular and trapezoidal enclosure has not been investigated the two parameters together considered in present work yet. Various inclination angle, heat source length and Rayleigh number, will be considered in this work.

2. Enclosure geometry:

Air with Prandtl number Pr = 0.7 filled. A two dimensional enclosure of length (L) and high (H) as shown in figures (1,a-b). The vertical inclined walls of the enclosure are maintained at same lower temperature (T_c), the bottom wall is heated isothermally and maintained at temperature (T_h) in two cases according to dimensionless length of the heat source as shown in figures (1,a-b), while the remaining walls are adiabatic.

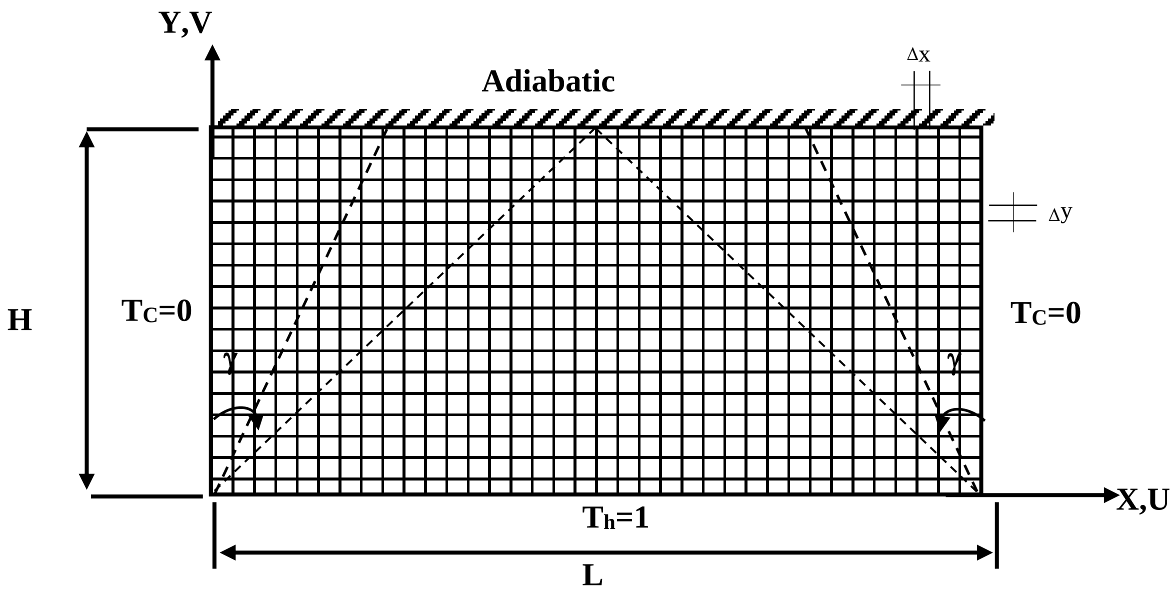

Figure 1 (a) Schematic diagram of the enclosure completely heated from below (W=L)

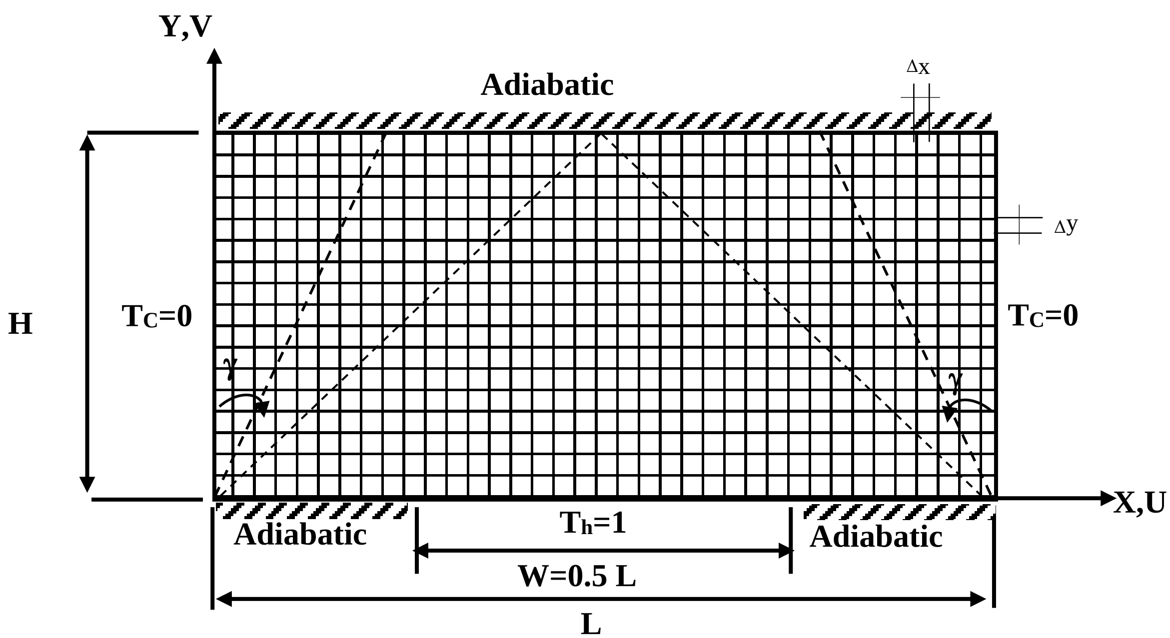

Figure 1 (b) Schematic diagram of the enclosure partially heated from below W=0.5L

3. Governing equations:

The governing equations include the equations of the continuity, momentum and the energy. The equations are as follows:-

1-The continuity equation for the steady and incompressible flow is given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

2-The following assumption, as steady flow, constant property of fluid and no heat generation are considered for the energy equation is:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \qquad (2)$$

3-The momentum equation in x and y directions are:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \qquad (3)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)-\rho\cdot g \qquad (4)$$

With Boussinesq approximations, the density is constant for all terms in the governing equations except for the bouncy force term that the density is a linear function of the temperature.

$$\rho=\rho_o(1-\beta\Delta T) \qquad (5)$$

By introducing equation 5 into equation 4 yield:

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+g\beta\Delta T \qquad (6)$$

The stream function (ψ) and vorticity (ω) in the governing equations are defined as follows:

$$u=\frac{\partial \psi}{\partial y}, \qquad v=-\frac{\partial \psi}{\partial x} \qquad (7)$$

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\omega \qquad (8)$$

The stream function formulation satisfies continuity equation and by substituting them in equation 1 yield.

$$\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}=-\omega \qquad (9)$$

By suitable manipulation, the pressure term in governing equations may be eliminated then,

$$\frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y}=\frac{\partial}{\partial x}\left(\mu\frac{\partial \omega}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial \omega}{\partial y}\right)+g\beta\frac{\partial T}{\partial x} \qquad (10)$$

Therefore the energy equation becomes [7].

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y}=\frac{\partial}{\partial x}\left(\frac{\mu}{\Pr}\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\frac{\partial T}{\partial y}\right) \qquad (11)$$

Dimensionless governing equations in streamline-vorticity form can be obtained via introducing the following dimensionless variables as follows:

$$X=\frac{x}{L}, Y=\frac{y}{L}, S=\frac{W}{L}, \phi=\frac{\psi \Pr}{\vartheta}, \theta=\frac{T-T_c}{T_h-T_c},$$

$$\Omega=\frac{\omega L^2 \Pr}{\vartheta}, \mathrm{Ra}=\frac{\beta g(T_h-T_c)H^3 \Pr}{\vartheta^2}, \ \Pr=\frac{\vartheta}{\alpha}$$

So the governing equations (stream function, vorticity and energy equations) can be written as [8]

Energy Equation

$$\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}=\left(\frac{\partial \phi}{\partial Y}\frac{\partial \theta}{\partial X}-\frac{\partial \phi}{\partial X}\frac{\partial \theta}{\partial Y}\right) \qquad (12)$$

Momentum Equation

$$\frac{\partial^2 \Omega}{\partial X^2}+\frac{\partial^2 \Omega}{\partial Y^2}=\frac{1}{\Pr}\left(\frac{\partial \phi}{\partial Y}\frac{\partial \Omega}{\partial X}-\frac{\partial \phi}{\partial X}\frac{\partial \Omega}{\partial Y}\right)-Ra\frac{\partial \theta}{\partial x} \qquad (13)$$

Continuity Equation

$$\frac{\partial^2 \phi}{\partial X^2}+\frac{\partial^2 \phi}{\partial Y^2}=-\Omega \qquad (14)$$

Boundary conditions
On the bottom wall

$$\phi = \Omega = 0, \quad S = 1, \quad \theta = 1 \;\&$$

$$\phi = 0, \quad 1/4 \le S \le 3/4 \quad \theta = 1 \qquad (15)$$

$$\phi = \Omega = 0, \quad S \le 1/4, \quad \frac{\partial \theta}{\partial Y} = 0 \;\&$$

$$\phi = 0, \quad S \ge 3/4, \quad \frac{\partial \theta}{\partial Y} = 0 \qquad (16)$$

On the top wall

$$\phi = 0, \qquad \frac{\partial \theta}{\partial Y} = 0 \qquad (17)$$

On inclined vertical walls

$$\phi = 0, \qquad \theta = 0 \qquad (18)$$

4. Numerical approach:

The governing equations for steady state, laminar flow, natural convection heat transfer in a two dimensional enclosure are solved using finite difference method based on Taylor series. In this method, all partial derivatives in governing equations are converted to forms that can be handled by computer program using FORTRAN language.

After applying the Taylor equation will get [8]:

$$\frac{\partial f}{\partial x} = \frac{f_{i+1,j} - f_{i-1,j}}{(1+a)\Delta x}, \quad \frac{\partial f}{\partial y} = \frac{f_{i,j+1} - f_{i,j-1}}{(1+b)\Delta y} \qquad (19)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{2}{\Delta x(1+a)}\left(\frac{f_{i+1,j} - f_{i,j}}{a\Delta x} - \frac{f_{i,j} - f_{i-1,j}}{\Delta x}\right) \qquad (20)$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{2}{\Delta y(1+b)}\left(\frac{f_{i,j+1} - f_{i,j}}{b\Delta y} - \frac{f_{i,j} - f_{i,j-1}}{\Delta y}\right) \qquad (21)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{2}{\Delta x(1+a)}\left(\frac{f_{i+1,j} - f_{i,j}}{a\Delta x} - \frac{f_{i,j} - f_{i-1,j}}{\Delta x}\right) \qquad (22)$$

The heat transfer from the walls of the enclosure can be obtained by using the integration of temperature gradient along the wall and then calculate the Nusselt number as following:

$$Q = \int_0^A \left(\frac{\partial \theta}{\partial x}\right)_{x=0,L} dy$$

$$Nu = \frac{Q_{conv.}}{Q_{cond.}} \qquad (23)$$

5. Results and discussion:

Numerical analysis of natural convection heat transfer and fluid flow was performed to obtain effects of the inclination angle of the vertical walls in a two dimensional enclosure and the effect of the heat source length which is located on the bottom wall of the enclosure, the inclined walls are kept at lower temperature, while remaining walls are adiabatic. Heat transfer and flow structure represented by stream function and isotherms line, and the effects of the parameters on the heat transfer represented by Nusselt numbers.

From figure (2) and figure (3) can notice the flow have two-cell symmetric about the mid-plane, this phenomena appear in all cases studies.

Figure (2) shows the effect of the length of the heat source on the structure of the flow and isotherm line for three values of the inclination angle. From this figure can be observe that the values of stream function and the temperature gradient increase with the increasing in heat source length, because the increasing in the heat source length causes increase in driving force (buoyancy forces). The figure also illustrates the effect of the inclination angle on the isotherm line and stream function. The increasing of the inclination angle cases decreasing in the stream function and temperature gradient because the increasing in the inclination of the vertical walls of the enclosure cases obstruction and decelerate of the flow subsequently the temperature gradient and stream function decreasing.

Figure (2) and figure (3) illustrate the effect of the increasing in Rayleigh number on the stream function and the isotherm line clearly from the figures can be observe that the increasing in Rayleigh number causes increasing in values of the stream function and the temperature gradient increase along the hot and cold walls of the enclosure this return to the increasing in Rayleigh number causes increasing buoyancy force therefore the stream function and the temperature gradient increases.

Figure 4 (A,B,C,D), figure 4 (B,D) shows the variation of the Nusselt with the angle of inclination (γ). The figure illustrates that the increasing in the inclination angle causes decreasing in the Nusselt number because the temperature gradient decreasing with the increasing in the inclination angle. Also from the figure 4 (A,C) we can see that the increasing in the Rayleigh number causes increasing in the Nusselt number, because the buoyancy forces increase with the increasing in the Rayleigh number, and the heat transfer by the natural convection increase due to the increasing in buoyancy forces subsequently the Nusselt number increase.

Figure 4 (A,B) and (C,D) shows the effects of the dimensionless heat source length (1,0.5) on the Nusselt number. It is observed that Nusselt number enhanced with the decreasing in dimensionless heat source length and the increase with the increasing in Rayleigh number

6. Conclusions:

The finite difference method is used to analyze the natural convection in two dimensional enclosure heated from below. The results show that:-

1-The decreasing in inclination angle causes increasing in value of stream function, temperature gradient and Nusselt number.

2-The decreasing in dimensionless length of the heat source causes increasing in the Nusselt number.

3-The increasing in Rayleigh number causes increasing in stream function, temperature gradient and Nusselt number.

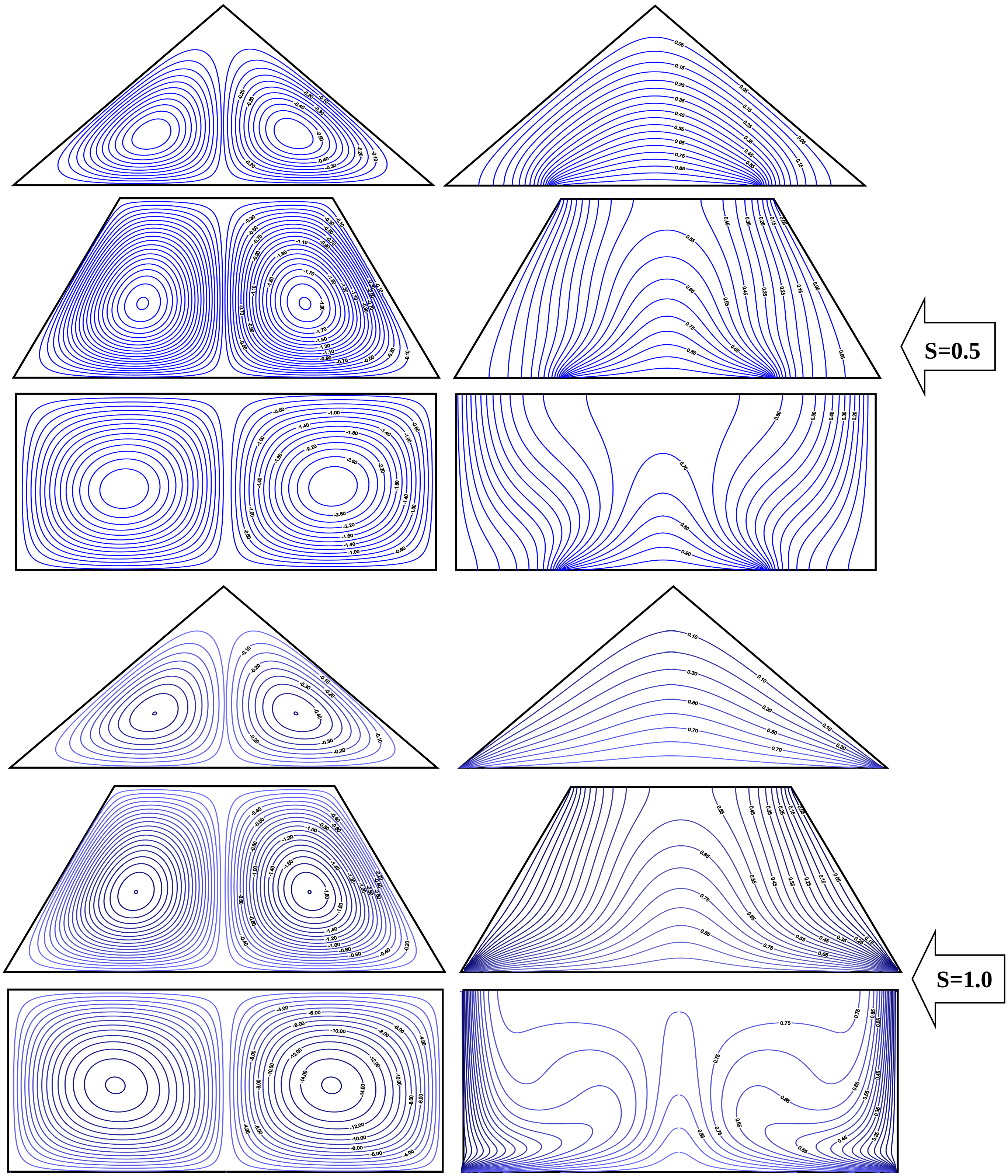

Figure (2) shows the effect of the inclination angle on isotherm line and stream function for (S=0.5 & S=1.0) and Rayleigh number $1*10^3$

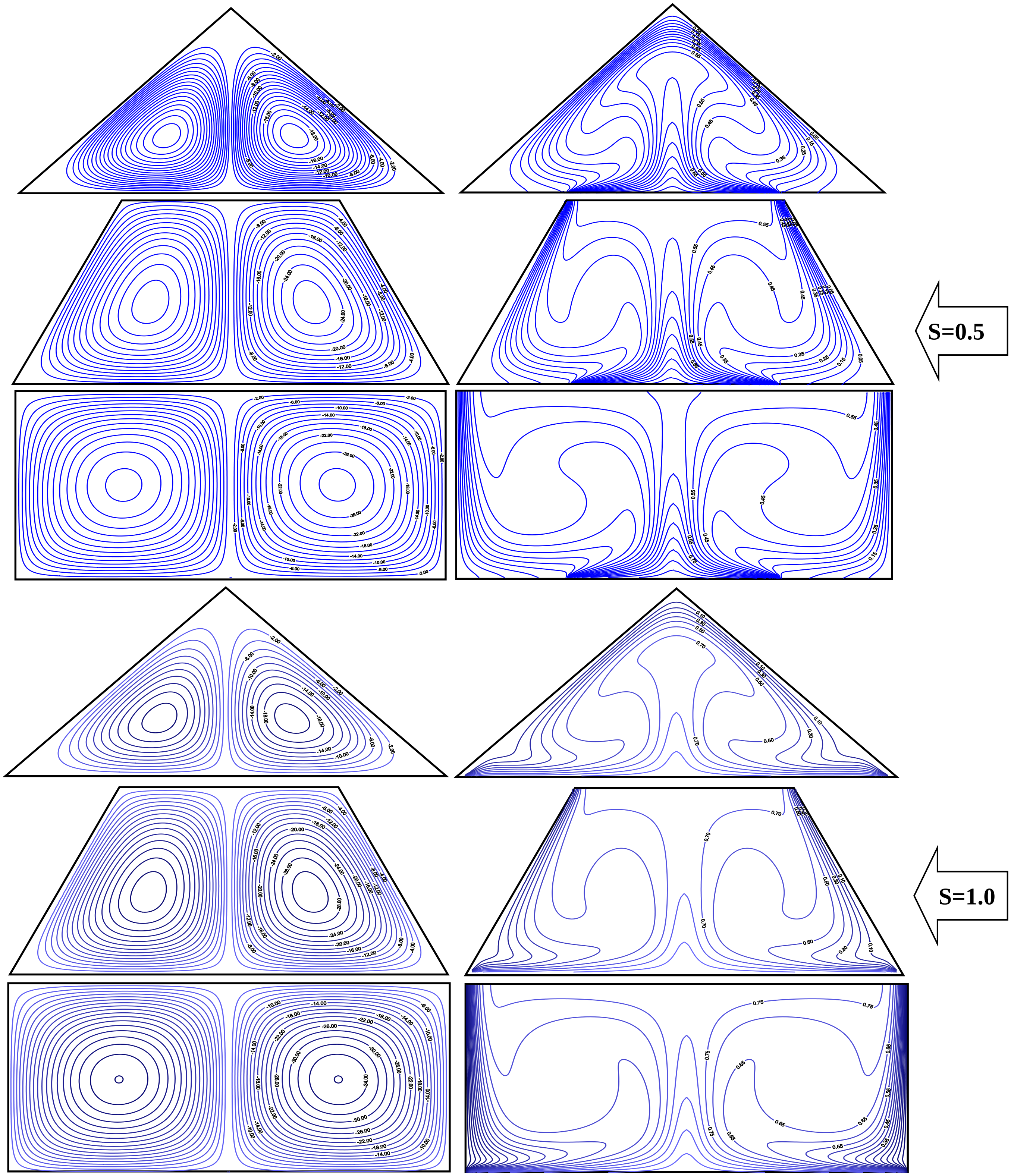

Figure (3) shows the effect of the inclination angle on isotherm line and stream function for (S=0.5 & S=1.0) and Rayleigh number $4*10^4$

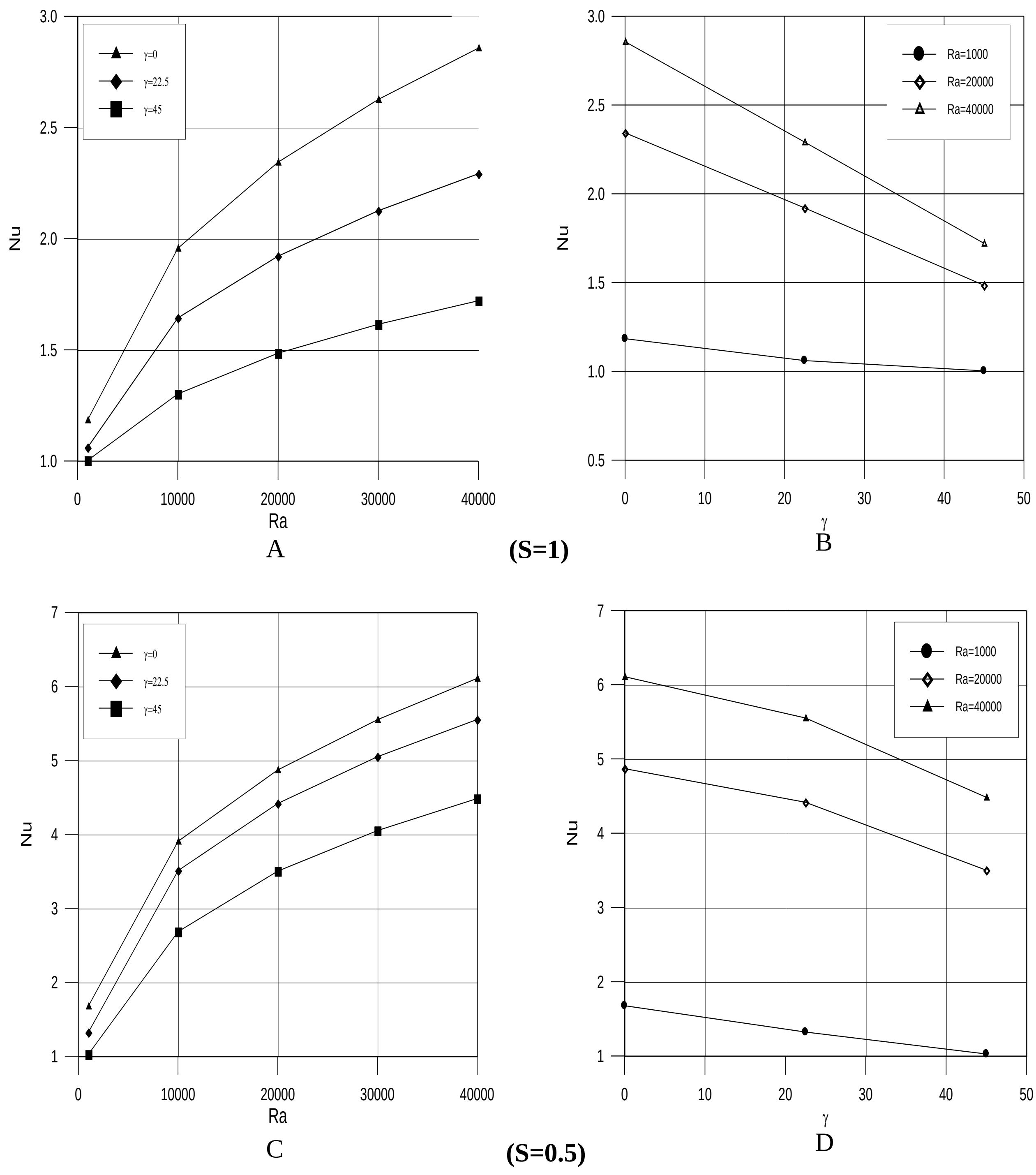

Figure (4,A-B-C-D) shows the effect of the Rayleigh number and the inclination angle on the Nusselt number for (S=1 & S=0.5)

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8. Nomenclature:

L	Width of enclosure (m)	ψ	Stream function
H	High of enclosure (m)	ω	Vorticity
Nu	Nusselt number	Ω	Dimensionless vorticity
Pr	Prandtl numbers	θ	Dimensionless temperature
Ra	Rayleigh number	ρ	Density (kg/m^3)
T	Temperature (°K)	ϑ	Kinematic viscosity (m^2/s)
U	Velocity in x direction (m/s)	γ	Angle of inclination of side walls (°)
V	Velocity in y direction (m/s)	α	Thermal diffusivity (m^2/s)
W	Heat source Length (m)	β	Thermal expansion coefficient (1/°K)
X	Horizontal distance (m)	**Subscript**	
Y	Vertical distance (m)		
S	Dimensionless Heat source Length	H	Hot
$\hat{\psi}$	Dimensionless stream function	C	Cold

تأثير زاويةِ ميل الجدران الجانبية على انتقال الحرارة بالحمل الطبيعي داخل حيز

رائد احمد علي **زياد محمد مجيد المخيول** **معن سعد الدين الدباغ**

مدرس **مدرس** **مدرس**

قسم الهندسة الميكانيكية – جامعة الموصل - العراق

الخلاصة :

في هذه الدراسة ، اجري التحليل العددي لانتقال الحرارة الطباقي وجريان المائع نتيجة التسخين من الأسفل داخل حيز ثنائي البعد ذو مصدر حراري متغير الطول . مليء الحيز بالهواء كمائع عامل. يحتفظ الجداران العموديان المائلان للحيز بدرجة حرارة منخفضة بينما يكون الجداران المتبقيان معزولين. كانت قيم عدد رالي من $(1x10^3 \le Ra \le 4x10^4)$ وزاوية الميل عند $(\gamma = 0^o , 22.5^o , 45^o)$ وطول مصدر التسخين اللابعدي عند $(S = 1 , 0.5)$. طبقت معادلات الاستمرارية والزخم و الطاقة للحيز وحلت باستخدام طريقة الفروقات المحددة. بينت النتائج بان معدل عدد نسلت يزداد بزيادة طول مصدر التسخين ويقل بزيادة زاوية ميل الجدارين العموديين.

<u>الكلمات الرئيسية:</u> انتقال الحرارة ، الحمل الطبيعي ، حيز ، مسخن من الأسفل ، ميل الجدران العمودية