FREE VIBRATION ANALYSIS OF A CANTILEVER CRACKED BEAM WITH SUBSTRUCTURE ATTACHMENT

Khaldon F. Breithe
M.Sc. Mech. Engineering
University of Anbar

Ghalib R. Ibrahim
M.Sc. Mech. Engineering
University of Anbar

Ahmed N. Uwayed
M.Sc. Mech. Engineering
University of Anbar

ABSTRACT

Free vibration analysis of a cracked cantilever beam with two types of additional substructure attachment is investigated using ANSYS program. The cantilever beam is used as a master structure with single substructure attachment in various locations (as 1-DOF mass attachment and 1-DOF mass-spring attachment) with influence of crack in different location and depths. The results for the changes of the natural frequencies of a cracked beam are compared with the results produced by Vahit et al [1]. So the same geometrical properties have been studied. In additional work a cracked beam carrying two types of substructure attachment are compared with the results of the beam without a crack and with multi crack depth. In all calculations the beam has a uniform cross-section and the crack was modeled by reduction in the modulus of the beam. The reducing effects of the cracked beam on the natural frequencies had been more apparent with the substructure attached to the beam in different situations. The effect of mass-spring substructure is larger than the effect of the attachment when modeled as mass substructure for the same mass, with 17% for the first natural frequency and 2% for the second and third natural frequencies. The results can be used to identify cracks in simple beam structure; cracks have a clearer decreasing impact on the natural frequencies.

KEYWORDS: Vibration, Ansys, Crack, Free

INTRODUCTION

Mechanical vibrations, long-term service or applied cyclic loads may result in the initiation of structural defects such as cracks in the structures. Accordingly, the determination of the effects of these deficiencies on the vibration safety and stability of the structures is an important task of engineers. The existences of crack in beam increases the local flexibility and modify its stiffness and damping properties. In view of that, the modal data of the structure hold information relating to the place and dimension of the defect. The influence of cracks on dynamic characteristics such as changes in natural frequencies, modes of vibration of structures has been the subject of many investigations. Cracked structures have been modeled by various methods in past researchers.

The effect of mass attachment on the transverse vibration characteristics of a cracked cantilever beam is investigated by Vahit and Haluk [1]. The governing equation for free vibrations of the cracked beam is constructed from the Bernoulli-Euler beam elements and the crack is represented by a rotational spring. The relative changes of the first three natural frequencies as a function of the location of the attached mass with two different distances location of crack had been studied. The reducing effects of the cracked beam on the natural frequencies had been more apparent with the mass attached to the beam in different situations.

Kisa and Gurel [2] proposed a numerical model that combines the finite element and component mode synthesis methods for the modal analysis of beams with circular cross section and containing multiple non-propagating open cracks. The model virtually divides a beam into a number of parts from the crack sections and couples them by flexibility matrices considering the interaction forces that are derived from the fracture mechanics theory.

Chondros et al. [3] developed a continuous cracked beam vibration theory for the lateral vibration of cracked Euler-Bernoulli beams with single-edge or double-edge open cracks. This continuous cracked beam vibration theory lead to a good approximation for the dynamic response to lateral excitation, as it can be easily extended to other vibration modes, geometries and boundary conditions and to coupled lateral and torsional vibration problems.

A continuous cracked beam vibration theory is used for the prediction of changes in transverse vibration of a simply supported beam with a breathing crack achieved by Chondros et al. [4]. The eigen frequency changes due to a breathing edge-crack are shown to depend on the bi-linear character of the system. The changes in vibration frequencies for a fatigue-breathing crack are smaller than the ones caused by open cracks.

Chen et al. [5] used the transfer matrix method for the dynamic analysis of a stepped beam with arbitrary multiple transverse open cracks. The reduction in bending stiffness due to the presence of transverse open cracks or abrupt changes of cross section is modeled by kind of massless rotational springs.

The vibrations characteristics of a cracked Timoshenko beam are analyzed using integrated finite element method and component mode synthesis, this project developed by Kisa et al. [6].

A simply supported beam is used as a master structure with unknown number of attachments (substructure) with two types of attachments models 1-DOF mass attachment and 1-DOF mass-spring attachment models, studied by Husam [7]. The effect of attachments on the master structure natural frequencies when modeled as (mass-spring substructure) is larger than the effect of attachments when modeled as (mass substructure) for the same attachment mass.

In this study, the effects of mass attachment on the free vibration of cracked beam carrying substructure 1-DOF mass attachment and 1-DOF mass-spring attachment models are discussed analytically using ANSYS program, which is one of the numerical techniques to analyze the structural element. The ANSYS program has many finite element analysis capabilities, ranging from a simple, linear, static analysis to a complex, nonlinear and transient dynamic analysis. The crack is modeled by reduction in the section modulus of the beam, and calculates of the transverse natural frequencies of the beam with crack and corresponding uncracked beam.

THEORETICAL MODEL

The effect of the attachment models 1-Dof mass attachment model and 1-Dof mass-spring attachment model on the first three natural frequencies of a cantilever beam with a crack as shown in Fig.(1) is given in form figures. Calculation in this study was carried out using ANSYS program via the following beam data: length (L=1m), height (h=0.01m), width (b=0.01m), young modulus (E=211GPa), Poisson ratio (ν=0.3), density (ρ=7860kg/m^3)

Three different crack location parameters Bc=Xc/L=0.2, 0.5 and 0.7 with a constant crack depth ratio a/h=0.4 are considered, as well as for Bc=0.5 different crack length are used as a/h=0.3,0.4 and 0.5. The mass parameter is kept constant as m/ρAL=0.3 with corresponding mass location (Bm=Xm/L) along the beam length.

The analysis guide manuals in the ANSYS documentation set describe specific procedures for performing analyses for different engineering disciplines

A typical ANSYS analysis has three distinct steps:

* Build the model
* Apply loads and obtain the solution.
* Review the results.

Assumptions and Restrictions: These are:

1- These structures have constant stiffness and mass effect.
2- There is no damping.
3-The structure has no time varying force, displacement, and temperatures applied (free vibration)

The element stiffness matrix is:

$$[K_l]=\begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3(1+\Phi)} & \frac{6EI}{L^2(1+\Phi)} & 0 & -\frac{12EI}{L^3(1+\Phi)} & \frac{6EI}{L^2(1+\Phi)} \\ 0 & \frac{6EI}{L^2(1+\Phi)} & \frac{EI(4+\Phi)}{L\ (1+\Phi)} & 0 & -\frac{6EI}{L^2(1+\Phi)} & \frac{EI(2-\Phi)}{L\ (1+\Phi)} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3(1+\Phi)} & -\frac{6EI}{L^2(1+\Phi)} & 0 & \frac{12EI}{L^3(1+\Phi)} & -\frac{6EI}{L^2(1+\Phi)} \\ 0 & \frac{6EI}{L^2(1+\Phi)} & \frac{EI(2-\Phi)}{L\ (1+\Phi)} & 0 & -\frac{6EI}{L^2(1+\Phi)} & \frac{EI(4+\Phi)}{L\ (1+\Phi)} \end{bmatrix} \quad (1)$$

The element mass matrix is:

$$[M_l]=(\rho A+m)L(1-\varepsilon^{in})\begin{bmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & A(r,\Phi) & C(r,\Phi) & 0 & B(r,\Phi) & -D(r,\Phi) \\ 0 & C(r,\Phi) & E(r,\Phi) & 0 & D(r,\Phi) & -F(r,\Phi) \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & B(r,\Phi) & D(r,\Phi) & 0 & A(r,\Phi) & -C(r,\Phi) \\ 0 & -D(r,\Phi) & -F(r,\Phi) & 0 & -C(r,\Phi) & E(r,\Phi) \end{bmatrix} \quad (2)$$

$$\Phi=\frac{12EI}{GA_S L^2} \quad (3)$$

Where:

$$A(r,\Phi)=\frac{\frac{13}{35}+\frac{7}{10}\Phi+\frac{1}{3}\Phi^2+\frac{6}{5}(r/L)^2}{(1+\Phi)^2}$$

$$B(r,\Phi)=\frac{\frac{9}{70}+\frac{3}{10}\Phi+\frac{1}{6}\Phi^2+\frac{6}{5}(r/L)^2}{(1+\Phi)^2}$$

$$C(r,\Phi)=\frac{(\frac{11}{210}+\frac{11}{120}\Phi+\frac{1}{24}\Phi^2+(\frac{1}{10}-\frac{1}{2}\Phi)(r/L)^2)L}{(1+\Phi)^2}$$

$$D(r,\Phi)=\frac{(\frac{13}{420}+\frac{3}{40}\Phi+\frac{1}{24}\Phi^2+(\frac{1}{10}-\frac{1}{2}\Phi)(r/L)^2)L}{(1+\Phi)^2}$$

$$E(r,\Phi)=\frac{(\frac{1}{105}+\frac{11}{60}\Phi+\frac{1}{120}\Phi^2+(\frac{2}{15}-\frac{1}{6}\Phi+\frac{1}{3}\Phi^2)(r/L)^2)L^2}{(1+\Phi)^2}$$

$$F(r,\Phi)=\frac{(\frac{1}{140}+\frac{1}{60}\Phi+\frac{1}{120}\Phi^2+(\frac{1}{30}-\frac{1}{6}\Phi+\frac{1}{6}\Phi^2)(r/L)^2)L^2}{(1+\Phi)^2}$$

$$r=\sqrt{\frac{I}{A}}$$

The element matrix when attaching mass is:

$$[M_l]=(\rho A+m)L(1-\varepsilon^{in})\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&0&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&0\end{bmatrix} \quad (4)$$

The equation of motion for an undamped system, expressed in matrix notation by using the above assumptions is:

$$[M]\{\ddot{w}\}+[K]\{w\}=0 \quad (5)$$

$$\{w\}=\{\varphi\}_i\cos\omega_i t \quad (6)$$

So that the equation of motion becomes:

$$-\omega_i^2[M]+[K]\{\varphi\}_i=0$$

or

$$[K]-\omega_i^{\,2}[M]=0 \quad (7)$$

Where: $\omega_i^{\ 2} = \frac{[K]}{[M]}$

RESULTS AND DISCUSSION

Most studies on the vibration analysis represent the crack by rotational spring such as Vahit et al. [1] and Murat et al. [2], with single type of substructure attachment. The current investigation uses ANSYS program to study the free vibration of cracked cantilever beam. The crack is modeled by reduction in the section modulus of the beam and two types of substructure attachment are used. The results have been compared with the Vahit &Haluk [1] results, in order to show the difference between the two methods used. So, the same geometrical properties of the beam are chosen as shown in Fig.(1) and the relative changes of the first three natural frequencies as a function of the location of attached substructure are shown, with second different type of substructure attachment to view the changes in the natural frequencies. The crack was located at three different distances from the fixed end of the cantilever beam. The results which one can see in Fig.(2) and Fig.(3) show a good agreement compared between this two researches.

In Figs.(3) through (5), the variation of the first three relative natural frequencies (normalized to the corresponding frequency ω_{no} of a beam without a crack and a point mass $= \omega/\ \omega_{no}$) are depicted as a function of the non-dimensional location parameter of the attached mass in the range of zero to one with absence of crack and three different crack location at B_c = 0.2, 0.5 and 0.7. It is known that the crack reduces the natural frequency and it is also evident from the figures that the first frequency reduction is higher when the crack are located near the fixed end while the second and third natural frequencies have been less affected.

Figs.(6) through (8), use second type of substructure as (1-DOF mass-spring attachment) along the beam length. The mass parameter is kept constant as $m/\rho gL = 0.3$ and the spring stiffness depending on the value of mass and the first natural frequency of uncracked beam without attachment mass as $2\pi f = \sqrt{\frac{k}{m}}$. The use of this type of substructure show high difference in reducing the relative natural frequency as compared with the first type of substructure (1-DOF mass attachment) and the variation of relative natural frequencies are very small if it used along the beam length.

Figs.(9) through (11), show the first three relative natural frequencies with three types of crack depth a/h = 0.3, 0.4 and 0.5 at the middle of the beam (B_c=0.5) with 1-DOF mass attachment. The natural frequencies of cracked beam are lower than those of corresponding uncracked beam. As the depth of the crack increase the difference in the reducing of natural frequencies is higher.

CONCLUSIONS

The main conclusions are summarized as:

1. Cracks have a clearer decreasing effect on the natural frequencies.
2. A crack near the free end will have smaller effect on the fundamental frequency than a crack closer to the fixed end.
3. The natural frequencies are almost unchanged when the crack is located away from the fixed end of the cantilever beam.
4. Natural frequencies of a cantilever beam depend on the location and depth of the crack.
5. The natural frequencies decrease when depth of the crack increases because of the bending moment along the beam.

6. The mass attachment is most effective at the free end of the beam for the first natural frequency, at B_m=0.4 point from the fixed end for the second natural frequency and at B_m=0.25 point for the third natural frequency.
7. A large effect of mass-spring substructure on the natural frequencies than mass substructure with constant changes from B_m=0.2 to the free end.
8. The results provide useful information for the determination of structural defects such as cracks and investigate the non-linear interface effects such as contact and impact when crack breathe.
9. The results can be used to identify cracks in simple beam structures, which give with two types of additional substructures in various situations.

REFERENCES

[1] Vahit Mermertas & Haluk Erol "Effect of mass attachment on the free vibration of cracked beam" 8th Int. Congress on Sound and Vibration,(2001).

[2] Murat Kisa & M. Arif Gurel "Model analysis of multi-cracked beams with circular cross section" Eng. Fracture Mech.73, 963-977, (2006).

[3] T.G. Chondros, A.D. Dimarogonas & J. Yao "A continuous cracked beam vibration theory" Jour. of Sound and Vibration 215, 17-34, (1998).

[4] T.G. Chondros, A.D. Dimarogonas & J. Yao "Vibration of a beam with a breathing crack" Jour.of Sound and Vibration 23, 57-67, (2001).

[5] Q. Chen, S.C. Fan & D.Y. Zheng "Natural frequency of stepped beam having multiple open cracks by transfer matrix method" Inter. Conference on computational methods, 15-17, (2004).

6] M. Kisa, J. Brandon & M. Topcu "Free vibration analysis of cracked beams by a combination of finite elements and component mode synthesis methods" computer & Structures 67, 215-223, (1998).

[7] Husam Mohammed Ali "Free vibration analysis of multi-body system" M. Sc. Thesis, University of Anbar –Mechanical Engineering, (2005).

[8] ANSYS theory references, 000855, Eight edition SAS, Ip. Inc.

NOMENCLATURE

A	area of the beam (m^2)
As	shear area (m^2)
E	young's modulus (N/m^2)
ρ	mass density (kg/m^3)
f	natural frequency
ω	frequency of the beam without any attachment (rad / sec)
ω_{no}	frequency of the beam without a crack and a point mass (rad / sec)
b	width (m)
h	height (m)
L	length (m)
I	area moment of inertia (m^4)
i	integer
k	spring constant (N / m)
m	attaching mass (kg)
M	mass of beam (kg)
$[M_l]$	the element matrix
Φ	global displacement
w	transverse deflections (m)
ωi	i^{th} natural frequency of vibration
[]	matrix
r	radius of gyration

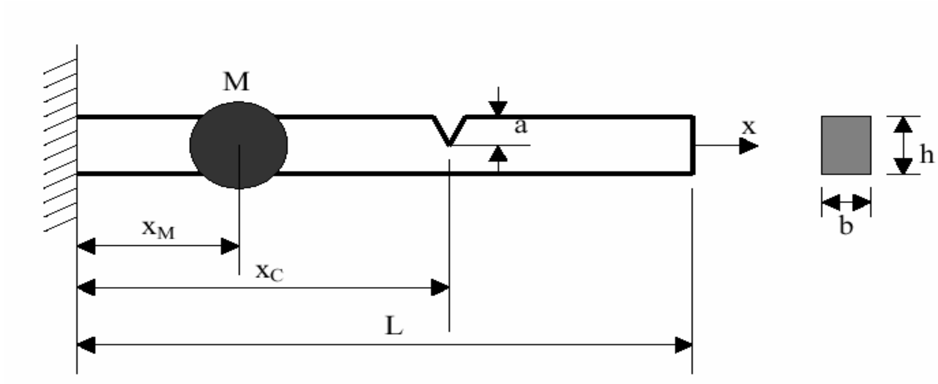

Figure (1): Cracked cantilever beam with 1-DOF mass attachment

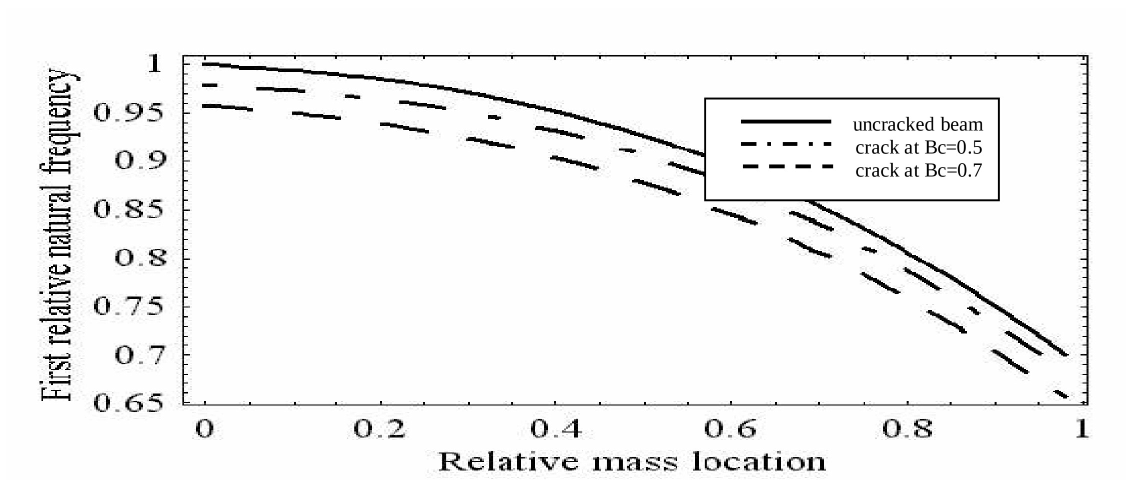

Figure (2): The relative changes of the first natural frequency as a function of the mass location for a cantilever beam. (a/h = 0.4, m/M = 0.3).[Ref.(1)]

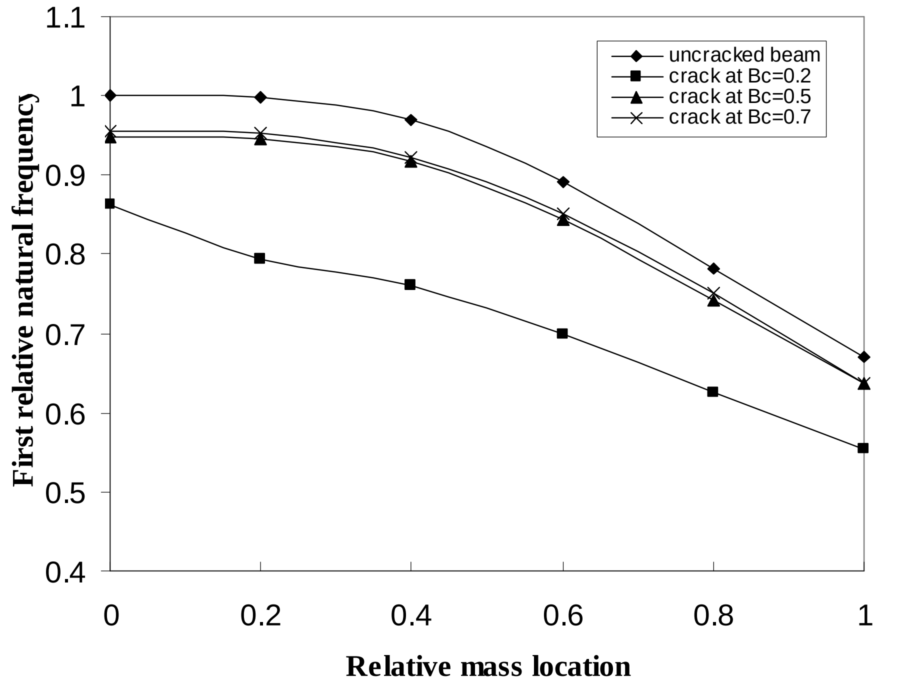

Figure (3): The relative changes of the first natural frequency as a function of mass location for a cantilever beam.

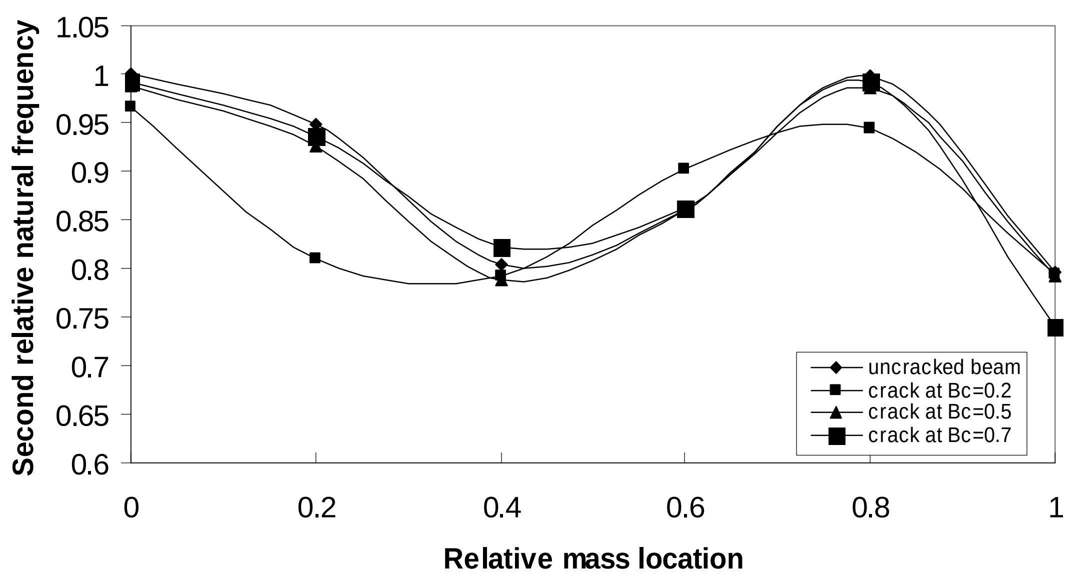

Figure (4): The relative changes of the second natural frequency as a function of mass location for a cantilever beam.

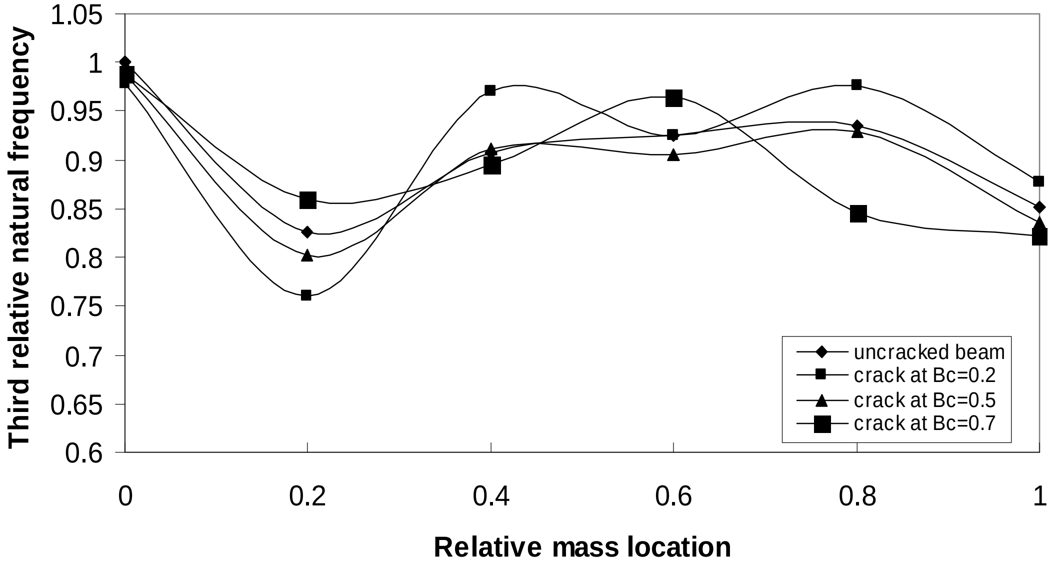

Figure (5): The relative changes of the third natural frequency as a function of mass location for a cantilever beam.

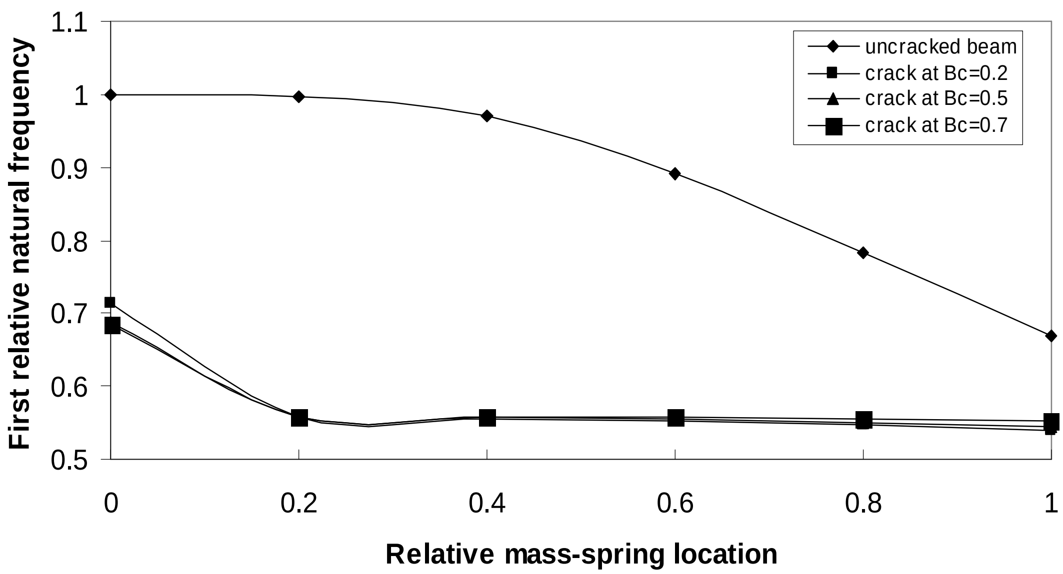

Figure (6): The relative changes of the first natural frequency as a function of mass-spring location for a cantilever beam.

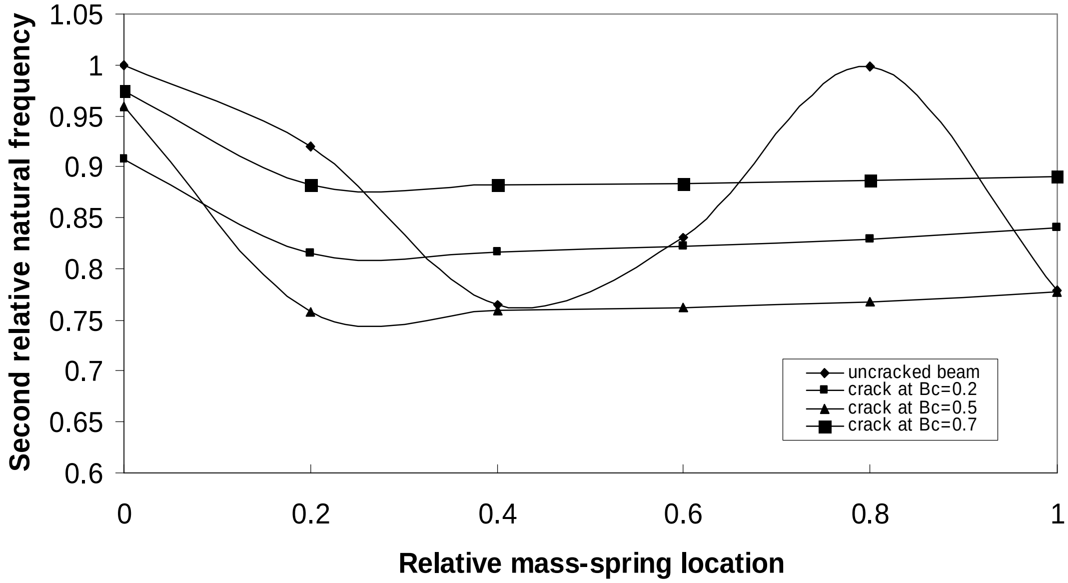

Figure (7): The relative changes of the second natural frequency as a function of mass-spring location for a cantilever beam.

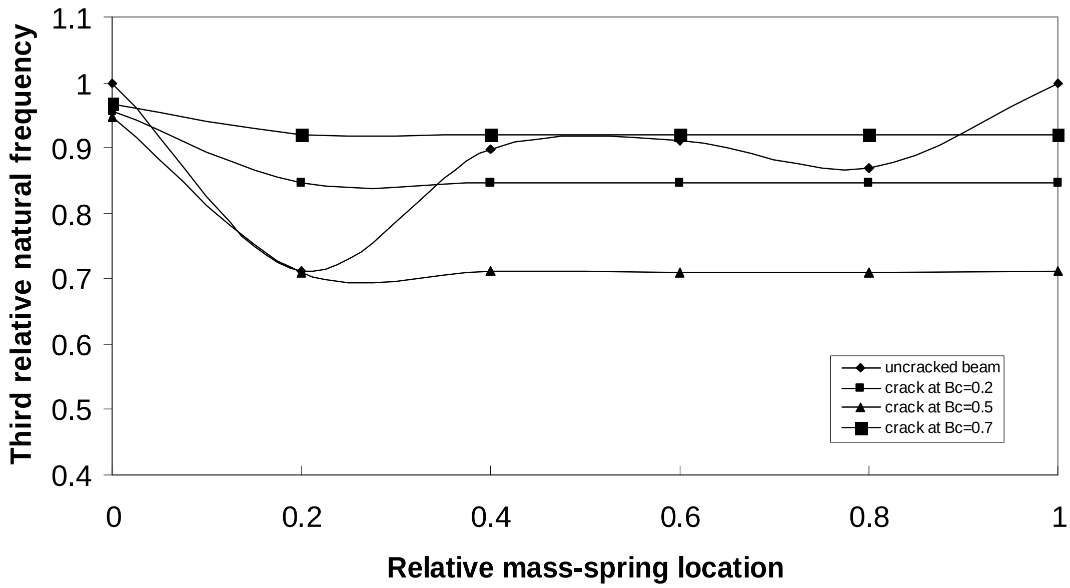

Figure (8): The relative changes of the third natural frequency as a function of mass-spring location for a cantilever beam.

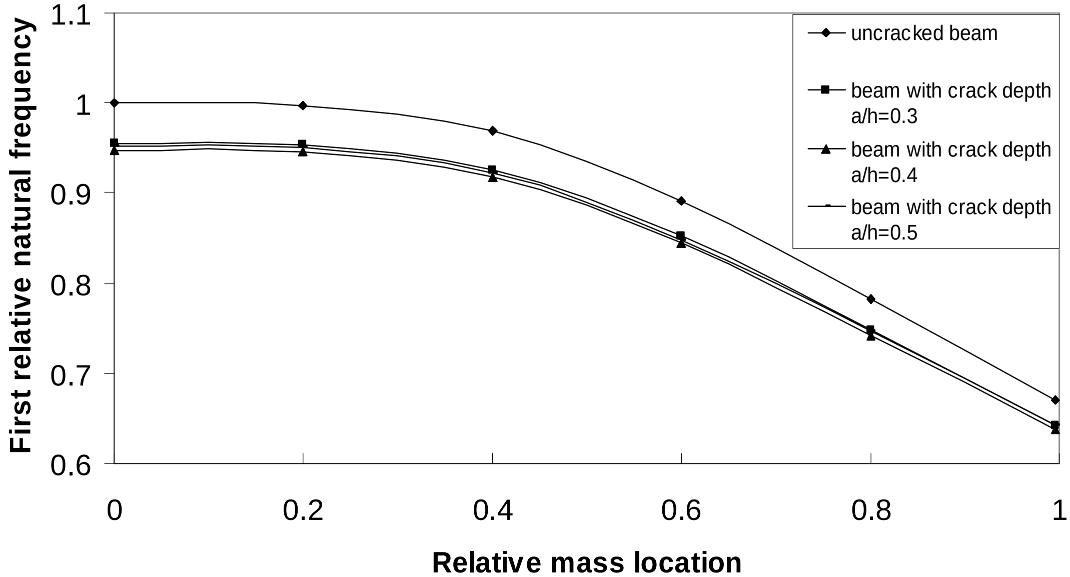

Figure (9): The relative changes of the first natural frequency as a function of mass location for a cantilever beam with different crack depth.

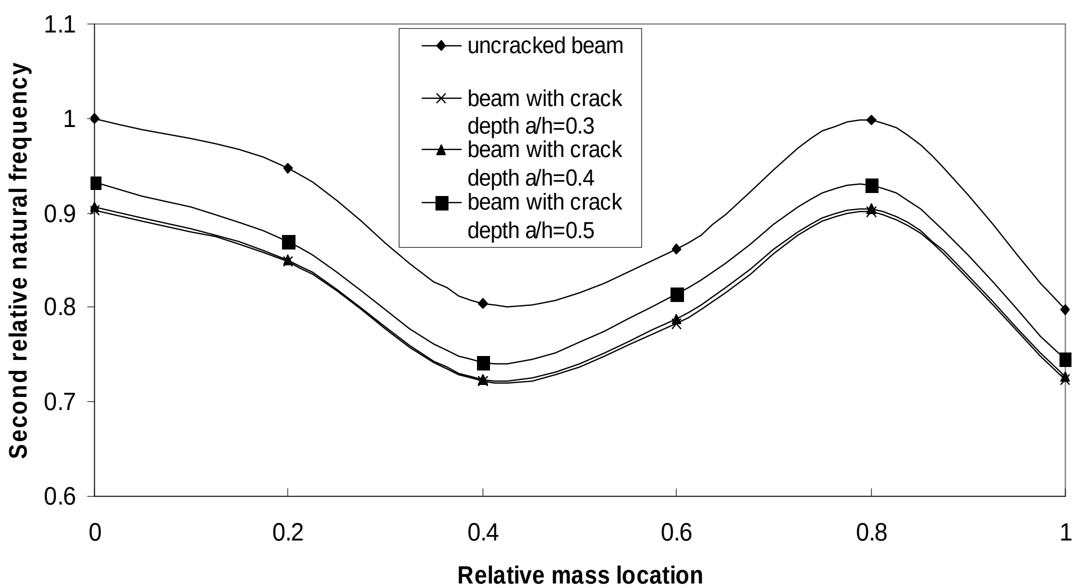

Figure (10): The relative changes of the second natural frequency as a function of mass location for a cantilever beam with different crack depth.

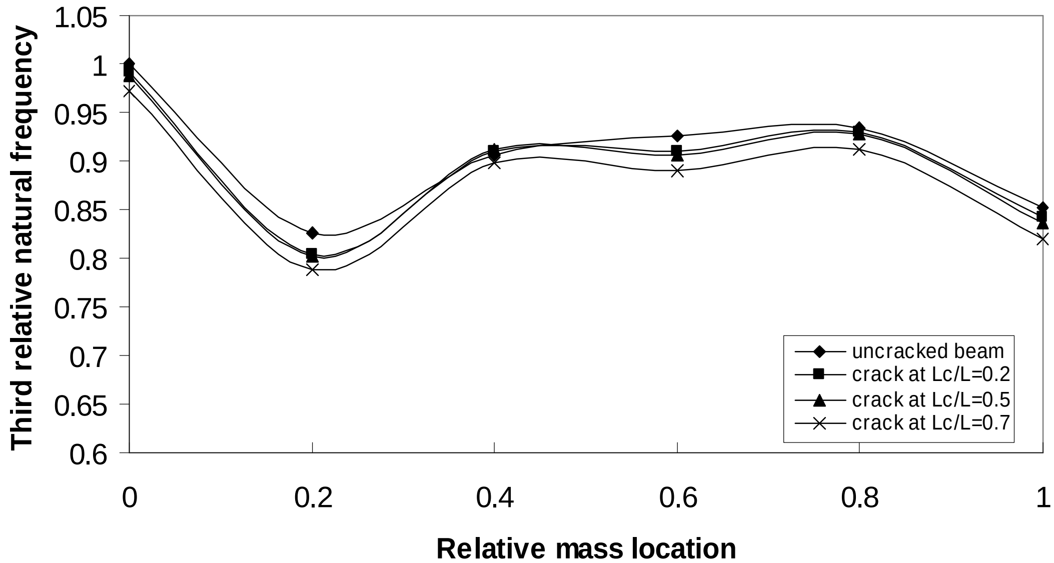

Figure (11): The relative changes of the third natural frequency as a function of mass location for a cantilever beam with different crack depth.

تحليل الاهتزاز الحر لعتبة مبنية ذات شق متعددة الاجسام

خلدون فاضل بريذع
ماجستير هندسة ميكانيكية
جامعة الانبار

غالب رزيك ابراهيم
ماجستير هندسة ميكانيكية
جامعة الانبار

احمد نوري عويد
ماجستير هندسة ميكانيكية
جامعة الانبار

الخلاصة

الاهتزاز الحر لعتبة ذات شق مبنية تم تمثيله بهيكل رئيسي مع استخدام نوعين من الهياكل الثانوية كنظام ذو درجة حرية واحدة (نظام كتلة و نظام كتلة - نابض) باستخدام البرنامج الهندسي المعروف ANSYS بوجود شق منفرد بمواقع واعماق مختلفة. استخدمت بعض النتائج للمقارنة مع نتائج البحث المقدم من قبل Vahit et.al.[1] لذلك تم استخدام المواصفات نفسها المستخدمة في ذلك البحث وبنفس الابعاد مع استخدام نوعين من الهياكل الثانوية بمواقع مختلفة على امتداد طول العتبة بحالة وجود الشق وبدونه وبعمق شق مختلف. اظهرت النتائج ان وجود الشق في العتبة يقلل الترددات الطبيعية وان قرب موقع الشق بالنسبة الى النهاية الثابتة يزيد في تقليل الترددات الطبيعية، كما وان الترددات الطبيعية تقل بزيادة عمق الشق الموجود في العتبة. كذلك ان استخدام هيكل ثانوي (نظام كتلة- نابض) له تاثير اكبر في تقليل الترددات من النوع الثاني المستخدم (نظام كتلة) وبنسب(١٧% للتردد الطبيعي الاول و٢% للتردد الطبيعي الثاني والثالث). يمكن استخدام النتائج المستحصلة لدراسة العتبات البسيطة وان وجود الشق يقلل الترددات وبالتالي يقلل حالات الصدم الحاصلة في الترددات الطبيعية.