DOA Based Minor Component Estimation using Neural Networks.

By

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Abstract.

Minor component analysis (MCA) of lower dimensional data is related to many signal processing applications. MCA strives to extract the "minor" direction in the data space where the variance of the data is minimal, identify the way for dimension reduction and data compression. In this paper neural networks are used to estimate the minor component of signal. This component is used to determine the Direction of Arrival Estimation (DOA) of incident signals. These signals are considered to be emitted from their emission sources .The neural networks knowing “Hebbian-networks” are used to estimate the minor component directions from signal subspace. Narrow band signals are considered here and strike an array composed of M sensors. Simulation results are introduced to shown the performance of the adaptive neural networks to estimate signal components, a comparison of the results obtained from classical method and MCA method, is presented which shows the performance of MCA over classical methods, to estimate exact signal direction from noise subspace.

1. Introduction.

Neural networks(NNs) have now been applied to a wide variety of real-world problems in many fields of application, the attractive and flexible characteristics of (NNs), such as parallel operation, learning by example, associative memory, multi-optimization and extensibility, make them well suited to the analysis of biological and other signals [1].

A neural network is information-processing system that has certain performance characteristics in common with biological neural network, NNs have been developed as a generalization of mathematical models of human cognition or biology, based on its properties [2].

Signal components extraction and estimation take a lot of research in different area, and different interest researchers. An extensive review of proposed methods of signal estimation of such maximum likelihood (ML) techniques, minimum norm method, multiple signal classification (MUSIC), can be found in[3,4]. The, minimum variance of capon, these classical methods are involve costly matrix inversions, as well as limitations of, estimation performance when the signal to noise ratio and number of samples are both too large, and the computational complexity. Principal and minor subspace computation is essential for many signal processing applications. A well-known tools for computing the principal and minor subspace of a data matrix is Oja’s rule [5]. Other method that involve simultaneous computation of principal and minor subspaces of a symmetric matrix [6], Weighted versions of these methods for joint computation of principal and minor components are also given.The proposed methods are derived from the optimization of certain objective functions over orthogonal constraints. In the literature, it is commonly admitted that minor subspace analysis (MSA) is a more difficult problem than principal subspace analysis (PSA), Recently, a new minor subspace tracker dedicated to time series analysis. This algorithm referred to as YAST[7], reaches the lowest complexity found in the literature, and outperforms classical methods in terms of subspace estimation.

The estimation performance and computational complexity form the central conflict in the direction of arrival approaches . To overcome this conflict the problem can be solved.using NNs.

The proposed Method in [8] concern a MCA to implement blind 2-channel equalization and discussed the null space method used to identify the non-minimum phase linear time variant channels. Coupled principal components analysis providing a framework of [9], for special class of learning rules where eigenvector and eigenvalues are simultaneously estimated in coupled update equation. Extraction of complex components using complex hebbian NNs was proposed by [2], which can be found in many areas for example, sensor array processing, beam forming in microphone array beam forming problem formulation, it shows that it closely resembles an MCA optimization problem in this field [10]. Independent component (ICA), principal component analysis(PCA), and MCA are used in work of [11] which involve multiple –source signal at an observation sensor and shared a common objective, that is to separate source signals. The aim of the work in[12] present an alternative method for estimating the direction of arrival (DOA), the algorithm exploit structural similarities between ESPRIT and the Tog-Xu-Kalaith method for blind channel equalization, and the result is an ESPRIT- Like algorithm for DOA estimation. manifold separation technique (MST) is a method for modeling the steering vector of antenna arrays of practical interest with arbitrary 2-D or 3-D geometry. It is the product of a sampling matrix (dependent on the antenna array only) and a Vandermonde structured coefficients vector depending on the wavefield only. This allows fast direction-of-arrival (DOA) algorithms designed for linear arrays to be used on arrays with arbitrary configuration[13].

2- Problem Definition.

Consider a uniform linear array of (M) sensors illuminated by (P) narrow-band signal (P<M) ,at the P'th snapshot, the output of the i'th sensor can be described by [14],

$$x = \sum_{i=1}^{d} \cos 2\pi p \Delta f(i) \exp(-j * (i-1) * 2\pi \Delta \cos(\pi - \theta(i))) \quad (1)$$

Where Δf is the frequency spacing of the sources, Δ is the space between two adjacent sensors, θ is the angel of arrival .Plane wave incident on uniform liner array and the spacing between two sensor (2 elements), resolve two closely separated sources [15]. The sources are assumed to be in far field thus the incoming waves are considered to be planner, the array output is corrupted by additive white noise which is assumed to be uncorrelated with sources signals, the incident signals are wave front [16]. In vector notation, the output of the array results from (p) complex signals can be written as[5,20]:

$$X(n) = C(\theta) S(n) + N(n) \quad (2)$$

where the vectors: **X**(n),**S**(n),**N**(n) are defined as;$\mathbf{X}(n) = (x_1(n) \; \ldots\ldots x_M(n))_{M\times 1}$, $\mathbf{S}(n) = (s_1(n) \; \ldots\ldots \; s_p(n))_{P\times 1}$, $\mathbf{N}(n) = (n_1(n) \; \ldots\ldots \; n_n(n))_{M\times 1}$

And the M × P matrix $\mathbf{C}(\theta)$ is defined as; $\mathbf{C}(\theta_i)$= [$c(\theta_1), c(\theta_2), \ldots\ldots, c(\theta_p)$] Moreover, this matrix is formed by using the following function.

$$c(\theta) = \exp[-j 2\pi \Delta\; i \sin\theta / v] \quad (3)$$

Where Δ, the spacing between two adjacent sensors and $\boldsymbol{v}$ the speed of light.
Equation 92) exploit the MCA modle from the received input data vector, expressed as linear combination.
The framework is to estimate the directions θ_i (for i=1,2,……,p) of source from available matrix (**X**). Assume the noise vector **n**(n) is stationary and of Gaussian process of zero mean and variance matrix $\sigma^2 I$, where σ^2 is unknown scalar [8].

3- Neural Minor Component Analysis (MCA).

A minor component analysis (MCA) strive to extract the principal components in the data space, where one seeks to find these directions that minimize the projection variance, thus paving the way for dimension reduction and data compression. These directions are the eigendirections corresponding to the minimum eigenvalue. The applications of minor component analysis arise in total–least square and eigenvalue-based spectral estimation methods. It allows the extracting of the first minor component from a stationary multivariate random process [15], based on the definition of cost function to be minimized under right constraints. The extraction of the last principal component is usually referred to as MCA[17]. MCA by neural network is a statistical signal processing technique investigated by Oja's [16], which allows the extraction of features from a given random signal or data stream.
In general, MCA is a statsirical method which can determine an optimal linear matrix W such that given an input vector x, the data in x can be combressed to form ;

$$y = \sum_{i=0}^{p} w_i x_i \qquad (4)$$

The direct extraction of Oja's first component, as the obtained learning algorithm is unstable, then recall from a simple but very interesting method for making this learning algorithm stable. For First minor component analysis (FMCA), it has now to find the weight vector (*W*) that minimizes the power of neurons output.
consider the problem of minimizing the following cost function [5];

$$C(W) = 1/2\,E_x[(W^T x_x)^2 / W + \lambda / 2(W^T W - 1)] \qquad (5)$$

With respect to the weight vector *W*, its gradient has the expression;

$$\frac{\partial C}{\partial \mathrm{w}} = \mathrm{E}_x\,[\mathrm{yx/w}] \quad + \lambda\mathrm{w} \qquad (6)$$

Where E denote expectation notation, and λ is constant.
The optimal multiplier may be found by vanishing $\mathrm{w}^{T}\frac{\partial \mathrm{c}}{\partial \mathrm{w}}$, that is by solving the gradient as.

$$\frac{\partial C}{\partial W} = E_x[y^2 / W] + \lambda W W^T = 0 \qquad (7)$$

The main point is to recognize that from an optimization point of view the above system is equivalent to,

$$\frac{\partial C}{\partial W} = E_x [y^2 / W] + \lambda\delta (WW^T - 1) \qquad (8)$$

With δ being an arbitrary constant. It is possible to prove that the FMCA converges to the expected solution providing that the constant δ is properly chosen. This is the way to compute the optimal multiplier to obtain the stabilized learning rule. The most exploited solution to the mentioned problems thus consists in invoking the discrete–time versions of FMCA rule, within this framework learning rule recast[2,14,20].

$$\Delta W = -\eta [YX - Y^2 W] - \eta\delta (WW^T - 1)W \quad , W(0) = W_0 \qquad (9)$$

where η is the learning rate parameter. It is a common practice to take η constant at a sufficiently small value which ensures good convergence in a reasonably short time, which represents the discrete time stochastic counterpart of FMCA rules. This equation may be easily implemented on a digital computer and exhibit minimal storage and computational requirements. In the first order the linear MCA will be[5,18] :

$$w_i(n+1) = w_i(n) - \eta\, y(n)[x_i(n) + y(n) w_i(n)] \qquad (10)$$

For multi output (neurons) the output y(n), of neuron j is produced in response to the set of input x_i (n), {i=1,2,3….m-1}, is given [14];

$$y_j(n) = \sum_{i=0}^{m-1} w_{ji(n)}\, x_i(n) \qquad (11)$$

The synaptic weight w_{ji}, is adapted in accordance with the generalized form of Hebbian [2,5,18] as;

$$\Delta w_{ji} = -\eta [y_j(n) x_i(n) + y_j(n) \sum_{k=0}^{j} w_{ki}(n)\, y_k(n)] \qquad (12)$$

Where $\Delta w_{ji}(n)$ is the change applied to the synaptic weight w_{ji} (n) at time n, and η is the learning-rate parameter. A compact form of GHA as;

$$\Delta w_{ji}(n) = -\eta y_j(n)[\tilde{x}_i(n) + w_{ji}(n) y_j(n)] \qquad (13)$$

where ;

$$\tilde{x}(n) = x_i(n) - \sum_{k=0}^{j-1} w_{ki}(n)\, y_k(n)$$

The oja's learning rule proved convergence to the weight vector (w_1), the first eigenvector of the covariance matrix R_{xx}, in practice the learning rate (η) is chosen to be small constant, in which case convergence is guaranteed with mean squared error in synaptic weight of order η [19]. The wave fronts received by M array element are linear combination of (d) incident wave fronts and

noise, the MCA begin with the model of the received input data vector, expressed as linear combination as in eq.(2).
Received vector x, and the steering vector c(θ) as vector in M dimensional space,and the input covariance matrix R_{xx}, can be expressed as;

$$\begin{aligned} R_{xx} &= E[XX^T] = E[WW^T]CC^T + E[NN^T] \\ &= R_{ss}CC^T + \delta^2_{noise} I \end{aligned} \qquad (14)$$

Where R_{ss} is the signal correlation matrix $E[WW^T]$. the number of incident signal d is less than the number of array elements M. this imply that the vectors of C are perpendicular to the eigenvector $V_{p+1},\ldots\ldots V_M$ corresponding to the noise.
Eigenvoctor of the variance matrix R_{xx} belong to one of two orthogonal subspaces. The orthogonal eigenvectors are the steering vectors of the recievd signala, and thus their direction of arrival can be determined.the MCA spetrum can be expressed as [21,22];

$$P(\theta) = \frac{1}{C(\theta)W_n W_n^T C^T(\theta)} \qquad (15)$$

Matrix WnW^T_n is a projection matrix onto noise subspace. For steering vectors that are orthogonal to the noise subspace, the denominator of eq.(15), will become very small, and thus the peaks will occur in P(θ) corresponding to the angel of arrival of the signal, from the expression in eq.(15), MCA is power spectral as function of ; θ , P(θ), C(θ),and R_{nn}.
Where ; $R_{nn} = \sum_{i=1}^{d} ww^T$, is the noise subspace.

4. Simulation Test of MCA.

This part about testing and discussing a result of MCA application by using simulation, to show performance of MCA as application to estimate the DOA of incoming signal and its parameters values. The signal is simulated with a steering angles, number of sources, and corrupted with different level of noise strength. Generalized hebbian algorithm is used as learning algorithm in neural network, with learning rate η of various value. A general test example is about one and two sources (d) used for this purpose, sinusoidal signal location is simulated at the far field at given direction degree, with normalized frequency were used. A uniform linear array (ULA) of snapshots (L), sensors (M) and spacing (Δ) between sensors, was used to collect the data, data are recorded using expression in Eq.(1). Testing were performed to show the effect of; snapshots, number of sensors, spacing of sensors, on the performance of DOA based MCA. Also learining rate parameter (η) effect is present to show the error convergence of neural algorithm at certain iterations.
Four test is performed include; 1- effect of snapshots variation (L= 5, 15, 30). 2- effect of sensors variation (M= 4, and 16), 3- effect of sensor spacing (Δ= 0.5λ, and λ) related to wavelength. 4- effect of error rate parameter (η) on the neural net convergence.
To asses the proposed method, it compared with classical approach known (MUSIC), in term of signal direction estimation with noise subspace.
Fig.(1) explain the effect of source variation of music method on an estimated angles of incoming multiple sources, were single source simulated at $\theta=60^0$, and two sources simulated at $\theta=80^0$, and $\theta=120^0$, number of snapshots (L=20), at certain level of noise, the distortion is clear shown due

to the effect of small number of snapshot and noise level, because the biases occurs in the spectral of signal.

Fig.(2) shows the effect of noise level on estimated DOA, using music method, when two incoming signals at θ= 80^o, and 100^o, with number of snapshots (L=20).

The effect of snapshots (L=5,15,30 respectively) on estimated DOA using MCA, of incoming single source at θ=80^o, M=16 sensors, and sensor spacing Δ= 0.5λ , is shown in **Fig.(3),** the effect of sensor variation (M=4,and 16) on MCA to estimate two sources of snapshots l=20, sensor spacing Δ= 0.5λ, and θ=80^o, 100^o is shown in **Fig.(4)**. **Fig.(5)** shows the effect of sensor spacing (Δ) on estimated DOA of incoming signals of θ=50^o, and θ=90^o,sensores M=8, and snapshots L=12.

The effect of learning rate parameter (η), on neural network convergence, is shown in **Fig.(6)**, at certain values of learning rate (η= 0.2, 0.01 ,and 1.2), GHA convergence is tested using different iteration values (10000, and 30000).

From the noise point view, MCA is better performance of DOA estimation at different level of noise over other methods, not only for noise side but also it estimate right DOA of multiple sources with variation of incoming angles. Snapshots test prove that MCA is able to estimate signals spectral even the number of snapshots is small, were the spectral of estimated signal is well better. Testing of sensor spacing imply MCA method estimate DOA of multiple sources at certain incoming angles, when Δ= 0.5λ, and from result, two targets are resolved with right angles, while increasing or decreasing this distance than above value, two sources are got spectral ambiguities and biasing of right angles of targets. Number of sensors is another performance measuring parameter used to measure the potential area of MCA in DOA estimation, it is clear from the results that varying the number of sensors in the array, the spectral is obvious, in that the peaks are sharp. When number of sensors decrease (less than 3) the spectral peaks is border, this yield that two sources are rarely resolved.

5. Conclusions.

A signal components estimation using minor component analysis and neural network algorithm, have been examined during this paper, by using simulated data.

Hebbian learning neural network algorithm, was introduced to estimate minimal signal components (MCA), which it used later to estimate DOA of simulated signals.

As the results of the intensive computer simulation performed during this study, the following conclusions can be drawn;

MCA is astatistical approach that is used to estimate the DOA for less number of samples (L=3) for signal and noise subspaces (dimensionality reduction), this method is powerful than traditional methods, because it estimate signal component directly from noise vector, while the rest estimate signal from vector of noise according to the criterion AKaike (AIC), consistency means convergence of the criterion to minimized number of sensors equal to number of sources (M=d).

MCA adapt to estimate DOA, for multiple sources, small size array (sensors = 4,16),and a range value of noise levels (variance). MCA tend to obtain a sharp spectral peaks of input signal when the snapshots are increased.

6. References.

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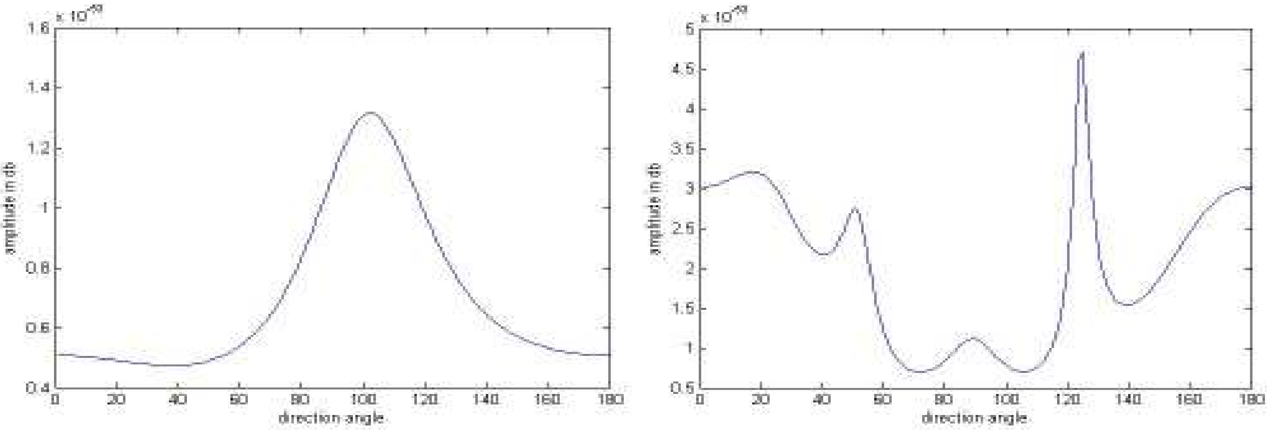
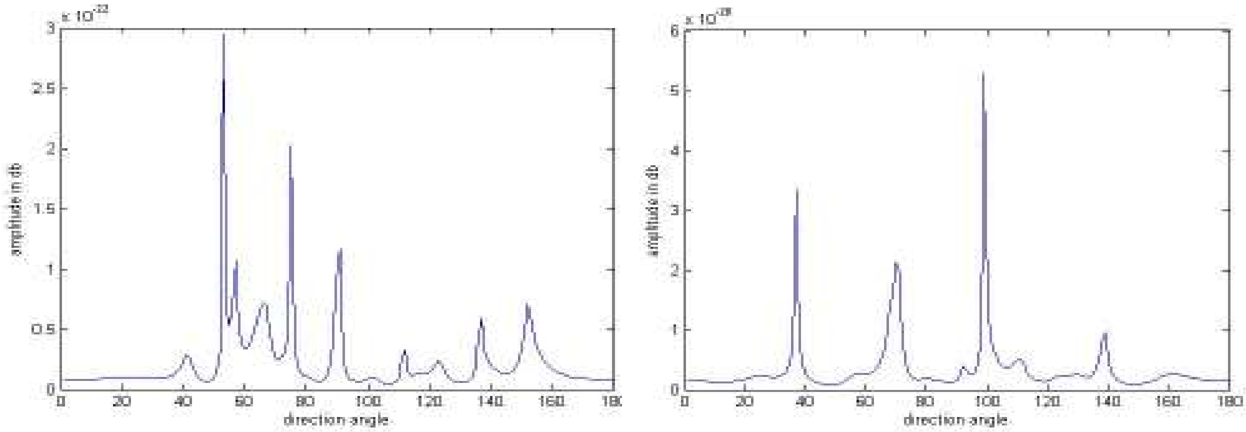


a b

Figure(1): incoming signal estimation of music method, snapshots (L=20) (a) one signal at 60^0,(b) two signals simulated at 80^0,120^0.



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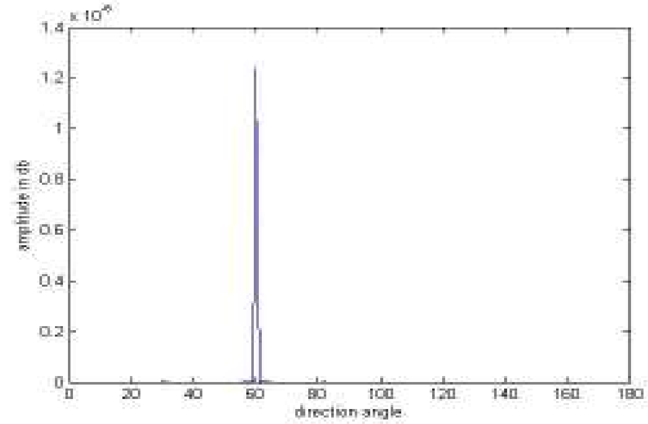
Figure(2): multiple signal estimation, with noise levels, snapshots (L=20 of music method,(a) two signal at 80^0 and 100^0,noise=0.1 (b) two signal at 80^0 and 100^0, noise=0.01.



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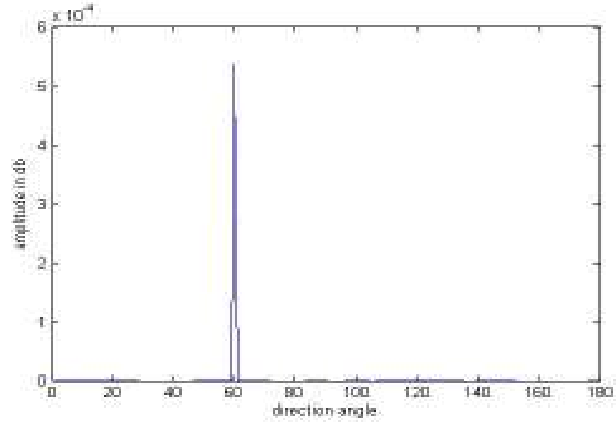

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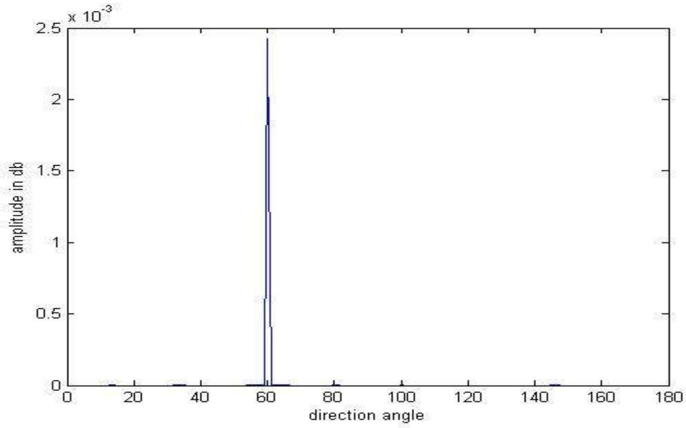

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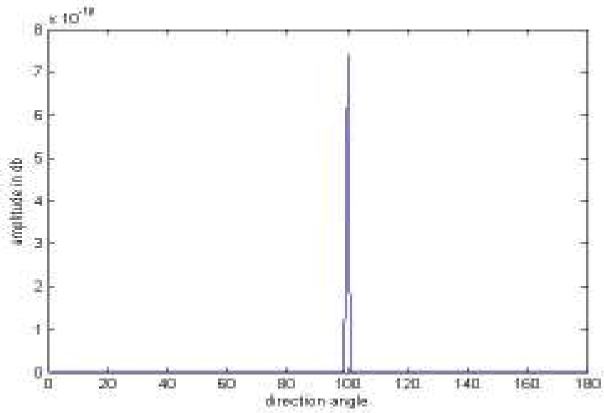
Figure(3): single source estimation for various snapshots, M=16, θ=60^{o}, Δ= 0.5λ (a) L= 5, (b) L=15, (c) L=30.



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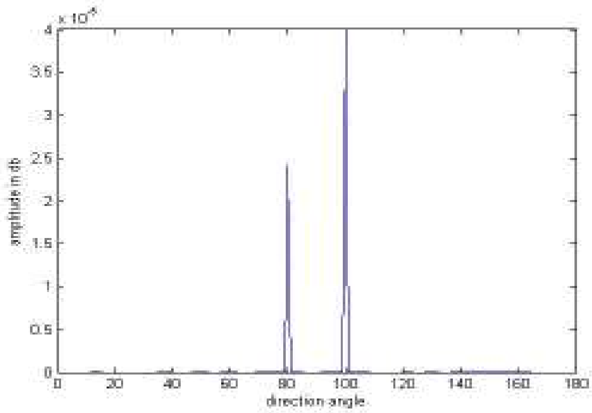

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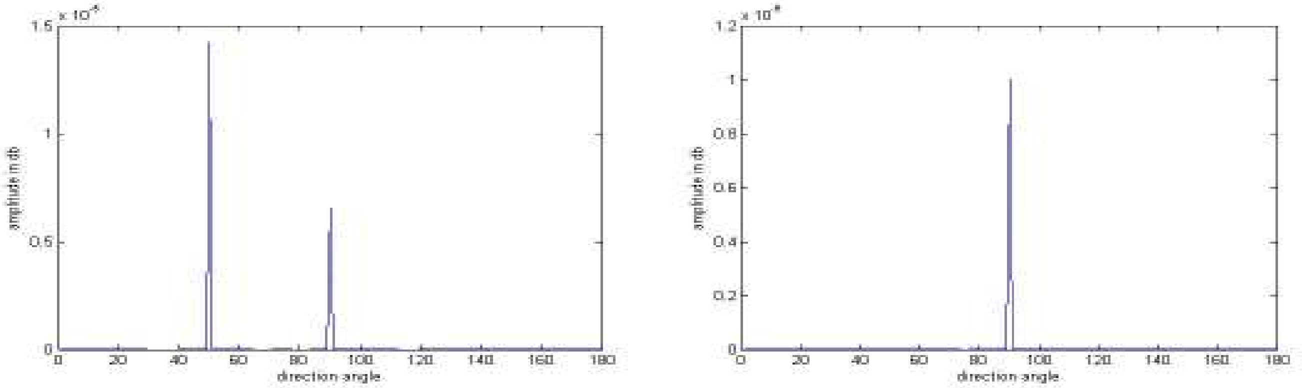
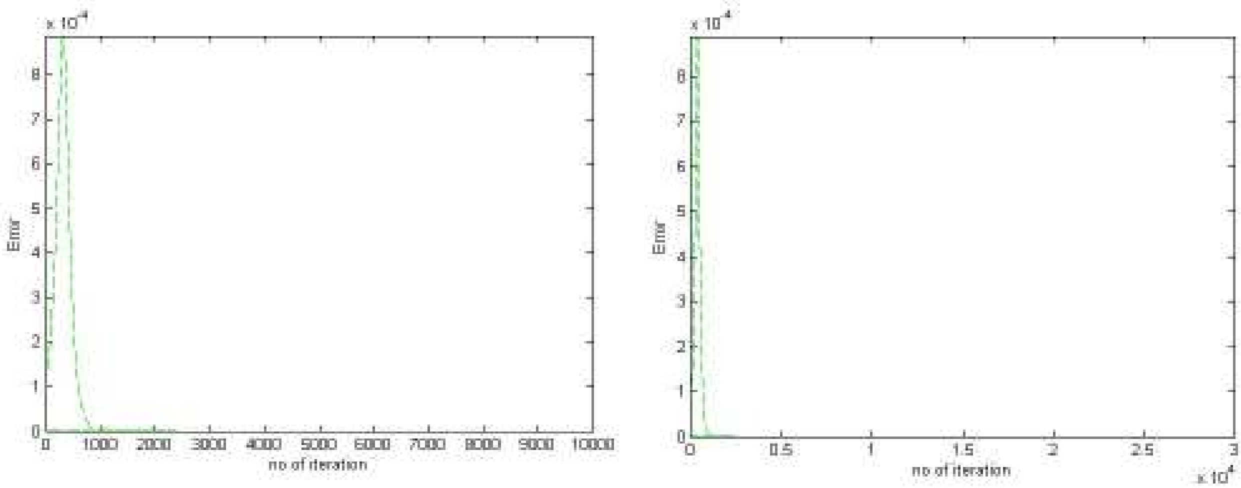
Figure(4): two signal estimation for various number of sensors, L=20, Δ= 0.5λ (a) two signal simulated when M=4, (b) two signal simulated when M=16.



a b

Figure(5): effect of sensor spacing of two signal simulated at angles $50^0$and 90^0, (a) Δ=0.5 λ, (b) Δ= λ.



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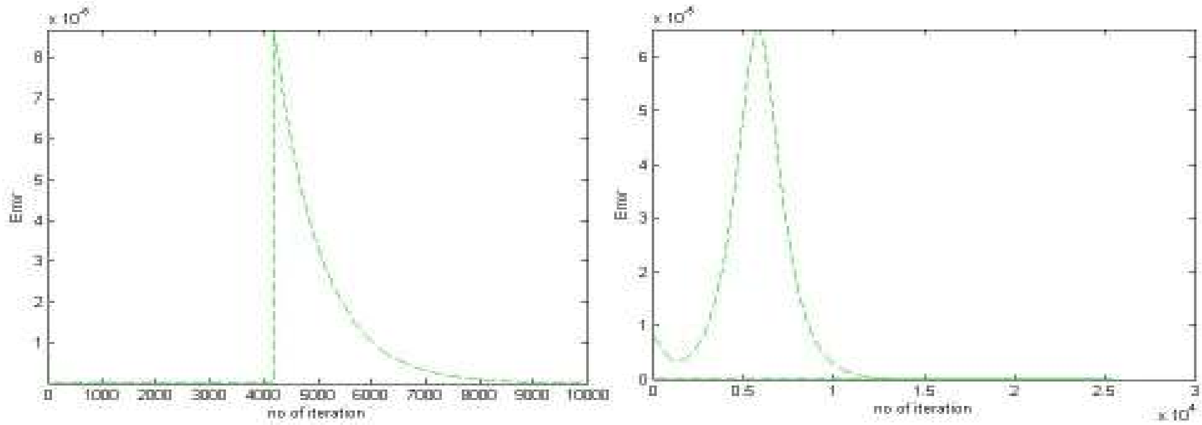

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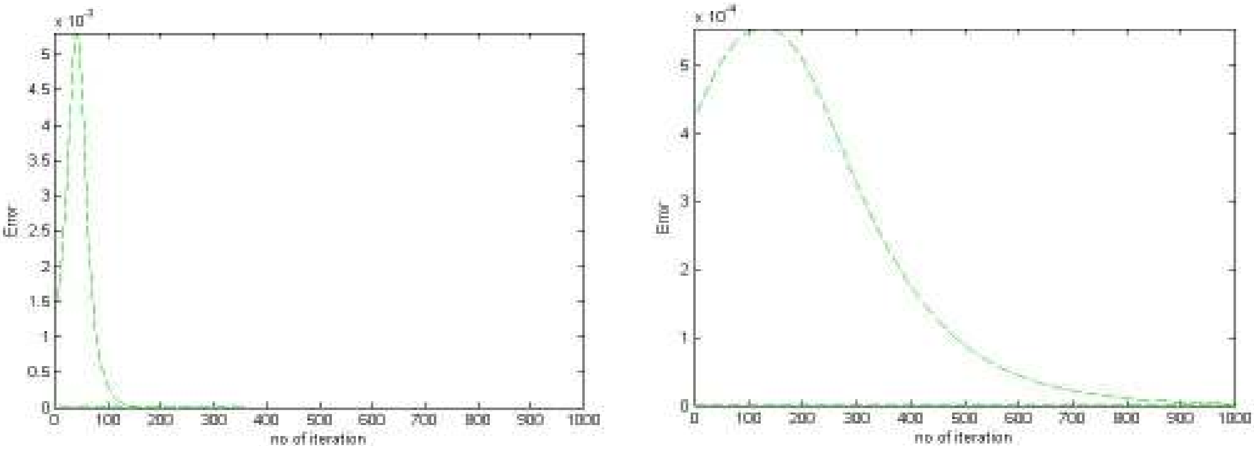

c

Figure(6): effect of learning rate (η) values on the convergence of the learning network, with various number of iterations, (a) η= 0.2 with iteration= 10000, 30000 (b) η=0.01 with iteration =10000,30000, (c) η=1.2 with iteration =10000,30000.

إيجاد اتجاه مسقط الإشارة اعتماداً على تخمين مُرَكِبات الإشارة الثانوية باستخدام الشبكات العصبية

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الخــــــلاصة

التحليل المكوّن البسيط (MCA) مِنْ البيانـاتِ البعديـةِ الأوطـأ يُتعلّـقُ بالعديد مِنْ تطبيقـاتِ معالجـةِ الإشـارات. يُجاهد (MCA) لإنتِزاع الإتّجاهِ "البسيطِ" في فضاءِ البيانات حيث أنَّ خلاف البياناتِ أقل ما يمكن، يُميّزُ الطريقَ لتخفيضِ البُعدِ وضغطِ البيانات. في هذاالبحث تم استخدام الشبكات العصبيةِ لتَخمين المكوّنِ البسيطِ للإشارةِ. هذا المكوّنِ يُستَعملُ لتَقْرير إتّجاهِ تقديرِ الوصولِ (DOA) مِنْ الإشاراتِ الساقطةِ. هذه الإشاراتِ تُبْعَثَ مِنْ مصادر إشعاعِها. أستخدمت مصدر الشبكات العصبيةَ المعروف ب "شبكات Hebbian" لتَخمين الإتّجاهاتِ المكوّنةِ البسيطةِ مِنْ الفضاءِات الثانويةِ البارزةِ.تم التركيز على إشاراتالحزمة الضيّقةِ ،وتَضْربُ صَفّاً متكوّن منM متحسّسات.من خلال نَتائِج المحاكاةِ،بينت ان أداءِ الشبكات العصبيةِ التكيفيةِ لتَخمين المكوّناتِ البارزةِ ،وتمت مقارنة النَتائِجِ التي تم الحصول عليها مِنْ الطريقةِ الكلاسيكيةِ(MUSIC) وطريقةِ MCA، حيث كان أداءَ MCA افضل من الطرقِة الكلاسيكيةِ، لتَخمين الإتّجاهِ البارزِ المضبوطِ مِنْ ضوضاءِ الفضاءَ الثانويَ.