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Modeling and Performance of a Wind Turbine

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ABSTRACT
The dynamics of wind turbine has to be studied carefully to avoid unpredictable outputs and to make sure that consistent and efficient power is supplied according to the load requirements. There is a great and urgent necessity to increase the efforts in the development of the researches of the renewable energy to decrease the dependency on the conventional ones. The objective of this research is to make a contribution to the ongoing wind turbine research in the area of modeling, which is the first step required for the control and implementation of wind turbines. The wind turbine transfer function is derived and its performance has been established using the MATHLAB Software. This research provided a different approach to wind turbine modeling methodology. The results of this research may be used in another step for completing the control process of the wind turbine.

Key Words: Wind Turbine, Modeling, Performance, MATHLAB.

نمذجة واداء توربين ريحي

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الخلاصة
من الضروري أن يتم العناية بدراسة ديناميكية التوربينات الريحية لتجنب النواتج (المخرجات) غير المتوقعة ولضمان توافق وكفاءة القدرة المجهزة وفقاً لمتطلبات الحمل. هناك حاجة كبيرة وملحة لزيادة الجهود المبذولة لتطوير البحوث في مجال الطاقة المتجددة وذلك لتقليل الاعتماد على الطاقات التقليدية. الغرض من البحث هو وضع اسهامة في تطوير بحوث التوربينات الريحية في مجال النمذجة (modeling) والتي تعتبر الخطوة الأساسية الأولى اللازمة للسيطرة على عمل التوربينات الريحية. تم اشتقاق دالة الانتقال (transfer function) للتوربين الريحي وتم حساب أدائه باستخدام (MATHLAB). يعطي هذا البحث مفهوم مختلف لطريقة نمذجة التوربين الريحي، النتائج التي تم الحصول عليها في هذا البحث يمكن استخدامها في الخطوة التالية لإكمال السيطرة على أداء التوربين الريحي.

1. INTRODUCTION

The main sources of electrical power are burning power generators which use the energy from non-renewable fuels to rotate a shaft connected to an electric generator [7]. These systems have seen vast improvements in the areas of efficiency, emissions and controllability because they have always been the primary power sources.

The oil crisis of 1973 caused an interest and great funds in wind energy and currently , with a great concern about the environment [1]. Its long lifespan, emission-free operation and low cost have made it more attractive compared to the other sources. It is also attractive for its job creation and the ease with which its technology can be transferred to developing countries. Since in most of the developing countries there are a great percentage of rural areas most of which are still far from electrical networks, so its suitable to use such type of technology (i.e., wind energy) for ensuring the electric demands of such areas [13].

In Iraq there are many region that have the range of wind speed of (3.5–5 m/s) which is considered useful for the application of wind energy [4] also these regions are more suited for the utilization ,due to their remoteness and relatively small and scattered population [3].

According to the decreasing of the conventional energy resources combined with their ecological consequences we must initiate a broad development program and make substantial funds for utilizing the renewable energy resources like solar and wind energy. The main focus of this research is the modeling of a wind turbine. The goal of this research is to have a significant contribution to the ongoing wind turbine research in the areas of modeling and control. A complete model will be developed and this will be ready to used as a basis for the design of a controller in another paper. For the mathematical model analysis, the rotor dynamics of the wind turbine were modeled using the Mat lab software

2. WIND POWER ASSUMPTIONS

The Rankine-Froude Actuator disc theory [7], [2] was used to define the wind profile over the rotor swept area for the analysis of wind turbine performance. The following assumptions apply:

1. Steady homogenous wind
2. No obstructions to wind flow either upstream or downstream of the rotor
3. Uniform flow speed at the disc
4. Wind flow passing through the disc is separable from the remaining flow by a well-defined stream tube.
5. Wind is incompressible
6. Wind flow passing through the disk is irrotational.

3. WIND POWER AVALIABLE

The kinetic energy, (U) of a packet of wind of the wind turbine is given by equation (1):

$$U = \frac{1}{2} m u^2 = \frac{1}{2} (\rho A x) u^2 \tag{1}$$

The power, (P_w) in the wind is the time derivative of the kinetic energy and is given by equation (2), which represents the total power available for extraction.

$$P_w = \frac{dU}{dt} = \frac{1}{2} \rho A u^2 \frac{dx}{dt} = \frac{1}{2} \rho A u^3 \tag{2}$$

It can be shown [7] that under optimum conditions, a tube of air approaching a wind turbine follows the profiles shown in figure (**1**).

The relations for the wind velocities and the cross-sectional areas in the following equation apply.

$$\left.\begin{aligned} u_2 &= u_3 = \frac{2}{3}u_1 \quad, A_2 = A_3 = \frac{3}{2}A_1 \\ u_4 &= \frac{1}{3}u_1 \quad, A_4 = 3A_1 \end{aligned}\right\} \tag{3}$$

As the wind passes over the turbine, the wind will lose power equal to the power extracted by the turbine, as given in equation (4)

$$P_{m,ideal} = P_1 - P_4 = \frac{1}{2}\rho\left(A_1 u_1^3 - A_4 u_4^3\right) = \frac{1}{2}\rho\left(\frac{8}{9}A_1 u_1^3\right) \tag{4}$$

This shows that for a given upstream tube-area (A1), an ideal turbine will extract (8/9) of the total power available in the wind. Since the upstream cross-sectional area is not physically measurable, in contrast the cross-sectional area of the wind turbine, the extracted power is usually conveniently expressed in terms of the wind turbine swept area (A2), which will be referred to as (A). Using equation (3), equation (5) can be obtained and shows that a turbine cannot extract more than (59 %) of the total power in an undisturbed tube of air with the cross-sectional area equal to the wind turbine swept area. This is known as Bertz theorem [7] and it limits all wind turbines to an efficiency of not more than (59 %).

$$P_{m,ideal} = \frac{1}{2}\rho\left(\frac{8}{9}\left(\frac{2}{3}A_2\right)u_1^3\right) = 0.59P_w \tag{5}$$

4. WIND POWE EXTRACTED

The fraction of power (P_m) extracted from the available power in the wind (P_w) by practical turbines is expressed by the coefficient of performance, (Cp). The power extracted (P_m) can then be expressed as shown in equation (6).

$$P_m = Cp\,P_m = Cp\left(\frac{1}{2}\rho A u^3\right) \tag{6}$$

The value of (Cp) varies with the wind speed, the rotational speed of the turbine, the rotor pitch angle, and the turbine blade parameters. It cannot exceed (0.59), as seen from equation (5). The tip speed ratio, (λ) is a variable that combines the effects of the rotational speed and the wind speed. It is defined as the ratio between the rectilinear speed of the turbine tip, (ωR), and the wind speed (u) as shown in Equation (7).

$$\lambda = \frac{\omega R}{u} \tag{7}$$

The parameter R is the maximum radius of the wind turbine's swept area.

5. VARIATION OF Cp WITH TIP SPEED RATIO AND PITCH ANGLE

Figure (**2**) illustrates the relationship between the coefficient of performance, (Cp), and the tip speed ratio, (λ), for various blade pitch angles (β) for a typical wind turbine. If the power performance of a wind turbine rotor is to be evaluated, its Cp(λ,β) curve might be obtained from the wind turbine manufacturer and a look-up table can be created to evaluate the coefficient of performance for each tip speed ratio and blade pitch angle. If the rotational speed, (ω) and pitch angle, (β), are known, ($\omega = \omega_o$ and $\beta = \beta_o$ for a constant rpm fixed pitch rotor), then the mechanical power output, (P_m), at any upstream wind speed, (u), can be found using equation (6).

Sometimes the full Cp(ω,β) data are not available so that equation (6) cannot be used directly in power performance evaluations. According to Justus [8], for any operation pitch angle, a good approximation to (Cp) as a function of the speed is found by using equations (8a) and (8b).

$$Cp = Cp_m = \left[1 - F\left(\frac{u_m}{u} - 1\right)^2 - G\left(\frac{u_m}{u} - 1\right)^3\right] \qquad u_e \le u \le u_R \tag{8a}$$

$$Cp = Cp_R = \left[\frac{u_R^3}{u^3}\right] \qquad u_R \le u \le u_F \tag{8b}$$

The coefficients (F) and (G) can be found using boundary conditions given by equations (9a) and (9b), which are derived from the fact that the coefficient of performance (Cp) is zero at the cut-in speed, and is (Cp_R) at the rated speed.

$$Cp\,(u_c) = 0 = Cp_m\left[1 - F\left(\frac{u_m}{u} - 1\right)^2 - G\left(\frac{u_m}{u} - 1\right)^3\right] \tag{9a}$$

$$Cp(u_R) = Cp_R = Cp_m\left[1 - F\left(\frac{u_m}{u} - 1\right)^2 - G\left(\frac{u_m}{u} - 1\right)^3\right] \tag{9b}$$

Combining these equations give the solution of (F) and (G) in a matrix form as given below:

$$\begin{bmatrix} F \\ G \end{bmatrix} = \begin{bmatrix} \left(\left(\frac{u_m}{u_c}\right) - 1\right)^2 & \left(\left(\frac{u_m}{u_c}\right) - 1\right)^3 \\ \left(\left(\frac{u_m}{u_R}\right) - 1\right)^2 & \left(\left(\frac{u_m}{u_R}\right) - 1\right)^3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 - Cp_R \end{bmatrix} \tag{10}$$

If (Cp_2) which is given in equation (8) is substituted into equation (6), the variation of mechanical power, (P_m), as a function of wind speed, (u), can be evaluated as shown below. Equation (11a) relates the input (wind speed, u) and the output (mechanical power, P_m) of the wind turbine rotor for ($u_c \le u \le u_R$). Equation (11b) is valid for ($u_R \le u \le u_f$).

$$P_m = P_R\left[\frac{Cp_m}{Cp_R u_R^3}\right]\left[(1-(F+G))u^3 + (2F-3G)u_m u^2 + (3G-F)u_m^3 - Gu_m^3\right] \tag{11a}$$

$$P_m = P_R \tag{11b}$$

For each pitch angle, this approximation method requires knowledge of the cut-in speed, (u_c), which is a function of the rotational moment of inertia of the rotor and the shaft, the maximum coefficient of performance, (Cp_m), the rated coefficient of performance, (Cp_R), the rated wind speed, (u_R), the wind speed at which (Cp_m) occurs, (u_m), and the rated power (P_m) of the wind turbine. Fortunately, (u_c), (u_R), (Cp_R), (Cp_m) and (P_R) are given for a wind turbine and the coefficients (F) and (G) can be found by applying the boundary conditions that $Cp(u_c)$ is zero, and (Cp_R) at rated speed.

The wind turbine rotor performance can also be evaluated as a function of the coefficient of torque (Cq). The wind turbine power, (Pw), is equal to the product of the torque, (T) and the rotational speed (ω). It follows that the torque coefficient, (Cq), can be related to the power coefficient, (C_P), through the relation shown in equation (12).

$$Cp(\lambda,\beta) = \lambda Cq(\lambda,\beta) \tag{12}$$

Therefore, manipulation of the torque coefficient using (λ) and (β) will result in manipulation of the power produced by the turbine. It is important to recognize the relationship between the aerodynamic torque (T_A) and the torque coefficient (Cq). Using equations (6) and (12), the aerodynamic torque, (T_A), that turns the rotor shaft is therefore represented by equation (13).

$$T_A = \frac{1}{2}\rho ARCq(\lambda,\beta)u^2 \tag{13}$$

The torque coefficient, (Cq), just like the coefficient of performance, (Cp), is a highly non-linear function of tip-speed ratio, (λ), and blade-pitch angle, (β). For variable pitch wind turbines, the coefficient of performance can be evaluated by the user for each pitch angle as outlined above and a surface representing the variation of (Cp) as a function of both (λ) and (β) can be generated. For a fixed tip speed ratio, the variation of (Cp) with the pitch angle can be approximated by fitting a curve to any Cp(β) data points that might be given for the wind turbine. This is done while paying attention to the pitch angle than gives maximum coefficient of performance.

6. WIND TURBINE OPERATION

In deriving a wind turbine mathematical model, a specific wind turbine had to be selected. This involved choosing the type of wind turbine operation. Variable-speed, constant-speed, pitch-control and stall-control were all analyzed and they are discussed previous.

The geometry and aerodynamics wind stream-33 [8] had been selected with wind turbine characteristics shown in table (**1**). With the wind turbine characteristics chosen, the next step was to derive the mathematical model of a pitch-controlled constant-speed wind turbine. Next section discusses the basic equations for such a wind turbine and shows the derivation of the mathematical model.

7. WIND TURBINE MODELING

7.1 Wind Turbine Plant Model

The wind turbine plant model was divided into two main parts. The first part was the wind turbine, which included a turbine rotor on a low-speed shaft, a gearbox and high-speed shaft. The inputs for this part of the plant were the wind speed and the blade pitch angle while the outputs were the high-speed shaft angular rotation and the mechanical power, (P_m). The second part was the electric generator whose input was constant angular rotation from the turbine plant and whose output was electrical power.

The following six steps show the sequence of events in the block diagram:

1. Wind and blade pitch angle are input into wind turbine plant, which causes the rotor to spin.
2. The low-speed shaft angular speed is fed back and compared to reference low speed shaft angular speed.
3. The angular speed error is input to the controller, which commands a change in blade-pitch angle.
4. A new blade pitch angle is applied to the actuator.
5. The actuator changes the blade pitch angle.
6. Wind at a new wind speed is input into the wind turbine plant and the six steps are repeated.

Although the goal of this control sequence is to maintain a constant angular speed and constant power, (P_m), only the angular speed is fed back to accommodate the wind speed fluctuations. This is because, controlling the angular speed automatically means that the aerodynamic torque, (T_A) that causes the rotation, is controlled and hence the extracted mechanical power, (P_m). This is derived from the fact that these three quantities, (P_m), (T_A) and (λ) are related by equation (14).

$$P_m = \omega . T_A \tag{14}$$

Therefore controlling (T_A) and (λ) to remain constant will cause the power (P_m) to remain constant as well. Throughout this thesis only angular speed control will be mentioned because its output profile will be closely related to that of the power (P_m) as will be seen later . The aerodynamic torque, (T_A), must be opposed by an equal and opposite load torque, (T_L), for the turbine to operate at steady speed. If (T_A) is greater than (T_L), the turbine will accelerate, while if (T_A) is less than (T_L), the turbine will decelerate. Equation (15) gives:

$$J_T . \omega_T = T_A . T_L \tag{15}$$

This mathematical description, where (J_T) is the equivalent combined moment of inertia of the rotor, gear reducer and both the low-speed and high-speed shafts. (T_L) is the mechanical torque necessary to turn the generator and was assumed to be a constant value derived from the wind turbine plant physical properties. The aerodynamic torque, (T_A), is represented by equation (13).

It is important to recognize the relationship between the power coefficient and the torque coefficient. Power extracted from the wind is shown in the equation (16) below:

$$P_A = \frac{1}{2}\rho A Cp(\lambda,\beta)u^2 \tag{16}$$

The turbine plant power, (P_m), is equal to the product of the aerodynamic torque, (T_A), and the rotational speed (ω) as shown in equation (14). The torque coefficient, (Cq), can be related to the power coefficient, (Cp), through the relation shown in equation (17).

$$Cp\,(\lambda,\beta) = \lambda\, Cq\,(\lambda,\beta) \tag{17}$$

Therefore, manipulation of the torque coefficient using (λ) and (β) will result in manipulation of the power produced by the turbine. Equation (17) is non-linear because it is a function of (Cp), which is highly nonlinear.

A traditional approach is to design a commonly used linear controller, such as proportional-integral-derivative (PID) and linearize the non-linear turbine dynamics about a specified operating point [11].

Assuming, ($T_A|_{OP} = T_L|_{OP}$) linearization of the equation (16) yields:

$$J_T\dot{\omega} = J_T\frac{\partial\dot{\omega}}{\partial u}\Big|_{OP}\Delta u + J_T\frac{\partial\dot{\omega}}{\partial\omega}\Big|_{OP}\Delta\omega + J_T\frac{\partial\dot{\omega}}{\partial\beta}\Big|_{OP}\Delta\beta \tag{18}$$

Simplifying equation (18) yields:

$$\dot{\omega} = \alpha\Delta u + \gamma\Delta\omega + \delta\Delta\beta \tag{19}$$

In this equation, ($\Delta\omega_T$), (Δu) and ($\Delta\beta$) represent deviations from the chosen operating point (ω_{TOP}), (u_{OP}) and (β_{OP}) respectively. The parameters (α), (γ) and (δ) are given by equations (20-22). Their detailed derivations are shown in appendix (**A**).

$$\alpha = \frac{\partial\dot{\omega}}{\partial u}\bigg|_{OP} = \frac{1}{2J_T}\rho A R u_{OP}\left[2C_q\big|_{OP} - \lambda_{OP}\frac{\partial C_q}{\partial\lambda}\bigg|_{OP}\right] \tag{20}$$

$$\gamma = \frac{\partial\dot{\omega}}{\partial\omega}\bigg|_{OP} = \frac{1}{2J_T}\rho A R^2 u_{OP}\frac{\partial C_q}{\partial\lambda}\bigg|_{OP} \tag{21}$$

$$\delta = \frac{\partial\dot{\omega}}{\partial\beta}\bigg|_{OP} = \frac{1}{2J_T}\rho A R u_{OP}^2\frac{\partial C_q}{\partial\beta}\bigg|_{OP} \tag{22}$$

The parameters (α), (γ) and (δ) represent the wind turbine dynamics at the linearization point. Their quantities depend on the wind speed and the partial derivatives of the coefficient of torque, (C_q) with respect to (λ) and (β) at the operation point. The magnitudes of (α) and (δ) show the relative weight of the effect wind speed (u) and the pitch angle (β), respectively, on the wind turbine angular speed. Equation (19) is the linear equation describing the wind turbine dynamics. Applying Laplace transforms yields:

$$\Delta\omega = \alpha\Delta u(s) + \gamma\Delta\omega(s) + \delta\Delta\beta(s) \tag{23}$$

Rearrangement the above equation yields to:

$$\Delta\omega = \left[\alpha\Delta u(s) + \delta\Delta\beta(s)\right]\frac{1}{s-\gamma} \tag{24}$$

After linearization about the chosen operating point for which the angular rotation speed is (ω_{TOP}), the wind speed (u_{OP}), and the pitch angle is (β_{OP}), equation (24) is a linear equation that describes the wind turbine dynamics in the Laplace domain. It represents the change in rotor speed output from the wind turbine. The inputs (Δu) and ($\Delta\beta$) represent deviations from the chosen operating points. Figure (4) shows the block diagram representation for equation (24). The wind turbine is therefore represented by the first-order transfer function $G_P(s)$ shown below:

$$G_P(s) = \frac{\Delta\omega(s)}{\alpha\Delta u + \delta\Delta\beta} = \frac{\Delta\omega(s)}{\left(\dfrac{\Delta T_A}{J_T}\right)} = \frac{1}{s-\gamma} \tag{25}$$

7.2 Wind Turbine Aerodynamic Data Specifications

The complete Cp(λ,β) data was not available for the Grumman Windstream-wind turbine. There were a few key data points relating (Cp), (λ), and (β), which were enough to evaluate the wind turbine performance. The available data is given in table (**2**) and it was obtained from the study that was conducted in [6].

8. MATHLAB SOFTWARE

The human limitations involved in compiling numerical and graphical data were formidable obstacles to implement more advanced qualitative or quantitative methods [9]. Computer platforms have reduced these obstacles with the using of the comprehensive mathematical software systems on personal computers. The approach formed in this research was treated using MATHLAB Software, and the results were found using this software and the program was listed in appendix (A).

9. RESULTS

Based on the available data, the linearized operating point was chosen to be ($\beta_{OP} = 9^o$), ($\lambda_{OP} = 7$), (u_{OP} = 7.5 m.s^{-1}) and (Cp_{OP} = 0.2). This operating point was chosen because is represents operation at relatively aerodynamically stable conditions. At the ($\beta = 3^o$), the wind turbine is operating at maximum (Cp) but it is also close to stalling conditions ($\beta < 3^o$). Therefore the blade pitch angle is restricted for lower operational blade pitch angles. This would also restrict the control scheme. At ($\beta = 12^o$), the blade pitch angle so much that the wind turbine can no longer extract any power from the wind. Therefore the operating point was set at ($\beta = 9^o$).

The coefficient of performance, (Cp) was evaluated at the linearized wind turbine operation blade pitch angle, (β), of (9^o). The variation of the coefficient of performance with the tip speed ratio was obtained using the method outlined previously. Figures (**5**) and (**6**) show the variation of (Cp) with wind speed, (u), and tip speed ratio, (λ), at the fixed pitch angle (β) of (9^o) for the Grumman Windstream-33 wind turbine based on the data in table (**2**). In comparison with the variation of (Cp) for other wind turbines [1, 10, 11], this method of approximation gave satisfactory results. Figure (**7**) shows the variation of the rotor mechanical power with wind speed, (u), at the fixed pitch angle (β) of (9^o) for the same wind turbine. At a wind speed of ($7.5\ m.s^{-1}$), which corresponds to a tip speed ratio of (7), the coefficient of performance and the mechanical power are regulated . The profiles obtained from Figures (**5**) to (**7**) were all obtained from the approximation method presented previously. All the profiles are comparable to general wind systems as found in the references. Figure (**8**) shows the variation of the coefficient of performance, (Cp), with the pitch angle, (β). This curve was derived from the three data points relating (Cp) and (β) in table (**2**) by fitting a third-order polynomial.

The coefficient of performance as a function of the pitch angle was derived using the three Cp(β) data points that were given at the operation (linearization) tip speed ratio of (7). A third-order polynomial was used to fit the data in order to derive the variation of the coefficient of performance, (Cp), with the pitch angle, (β). The constants at the operation point, the derived Cp(λ,β) curves and the relation between (Cp) and (Cq) were used to calculate the wind turbine dynamic constants (α), (δ) and (γ). Table (**3**) shows the values obtained for these parameters.

The value of (α) is positive because any increase in the wind speed causes the rotor to accelerate. However, (δ) is negative because increasing the pitch angle leads to a decrease in the rotor speed. For the wind turbine plant to be stable, the value of (γ) should be negative to make the pole of the wind turbine plant transfer function fall on the left hand side of the s-domain.

It should be noted that linearization is valid only when the wind turbine is to be analyzed at near linearization conditions. At the specified operating points, all the wind turbine performance parameters fall on the derived Cp(λ,β) curves. Therefore, the wind turbine dynamic model was fairly accurate for simulations at or near the operating conditions.

10. CONTRIBUTIONS

This research contributed a method of wind turbine performance estimation, modeling, linearization and control. The specific contributions are:

1. The wind turbine model was derived using a linearization technique coupled with the approximation of the wind turbine dynamics. Given that there was no readily available data for a straightforward data reduction procedure, the method developed by Justus [8] was used and it proved to be successful based on the output profiles that were obtained for all the simulations. All of them were comparable to the profiles of other wind turbines that have been studied [7]. This was a different and useful approach to wind turbine modeling.
2. The human limitation involved in compiling numerical and graphical data were formidable obstacles to implementing more advanced qualitative or quantitave methods. In this work the using of the comprehensive mathematical software system MATHLAB reduces these obstacles and its obvious that the software used in this work is a great tool to deal with such studies.

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Appendix (A): MATHLAB Code for Variation of Cp with u and λ

```
%Cq and Cp curve as a function of Lambda
close all;
clear all
clc;
J_T=1270; %Moment of inertia
pr=20000; %Rated Power
r=5; %radius of the rotor
rho=1.225; %density
uc=6.5; %cut in speed
ur=11.7; %rated speed
uf=24; %Furling speed
omega=105; %angular speed
cpm=0.2596; %maximum cp
lamdam=5; %tip speed ratio at maximum cp
um=11; %wind speed at maximum cp
cpr=pr/(0.5*rho*ur^3*r^2*pi);
```

```
% Operation or linearization point
lamb_op=7; %operation tip speed ratio
u_op=7.5; %operation wind speed
cp_op=0.2; %operation Cp
cq_op=cp_op/lamb_op; %operation Cq
% Solve the A and B coefficients
A_B=inv([((um/uc)-1)^2 ((um/uc)-1)^3;((um/ur)-1)^2 ((um/ur)-
1)^3])*[1;1-cpr/cpm];
A=A_B(1,1);
B=A_B(2,1);
%Find the Cp and P variation
uu=[uc:0.01:uf]';
u=[uc:0.01:ur]';
u1=[ur+0.01:0.01:uf]';
for i=1:size(u)
for k=1:2;
beta(k)=0;
cp(i,k)=cpm*(1-A*((um/u(i))-1)^2-B*((um/u(i))-1)^3);
p(i,k)=0.5*rho*pi*r^2*cp(i,1)*u(i)^3;
lamb(i,1)=r*omega*2*pi/(60*u(i));
cq(i,k)=cp(i,k)/lamb(i,1);
end
end
for k=1:size(u1)
j=k+521;
for l=1:2
beta(l)=0;
cp(j,l)=(cpr*ur^3)/u1(k)^3;
p(j,l)=0.5*rho*pi*r^2*cp(j,1)*u1(k)^3;
lamb(j,1)=r*omega*2*pi/(60*u1(k));
cq(j,l)=cp(j,l)/lamb(j,1);
99
end
end
% Plotting time!!!!!
figure(1)
plot(uu,cp(:,1)),grid;
title('Figure 1: Graph of Coefficient of performance, cp as a
function
of wind speed');
xlabel('wind speed (m/s)');
ylabel('C_p')
figure(2)
plot(lamb,cp(:,1)),grid;
title('Figure 2: Graph of Coefficient of performance, cp as a
function of tip speed ratio');
xlabel('tip speed ratio');
```

```
ylabel('C_p')
figure(3)
plot(uu,p(:,1)),grid;
title('Figure 3: Graph of Rotor Mechanical Power as a function of
wind speed u');
xlabel('wind speed (m/s)');
ylabel('Rotor Mechanical Power')
figure(4)
plot(lamb,cq(:,1)),grid;
title('Figure 4: Graph of Coefficient of torque, cq as a function
of tip speed ratio');
xlabel('tip speed ratio');
ylabel('C_q')
figure(5)
surf(beta, lamb, cp);
AXIS([-2 2 1 15 0 .6])
title('Figure 5: Graph of beta, lamda versus the coefficient of
performance')
xlabel('beta')
ylabel('lamda')
zlabel('Cp')
figure(6)
surf(beta, lamb, cq);
AXIS([-2 2 1 15 0 .06])
title('Figure 6: Graph of beta, lamda versus the coefficient of
Torque');
xlabel('beta')
ylabel('lamda')
zlabel('Cq')
dcq_dlamb=-(cpm/lamb_op^2)*(1-A*((um/u_op)-1)^2-B*((um/u_op)-
1)^3);
%(dCq/dlambda)
alpha=(1/J_T)*0.5*rho*pi*r^3*u_op*(2*cq_op-lamb_op*dcq_dlamb)
%alpha
gamma=(1/J_T)*0.5*rho*pi*r^4*u_op*dcq_dlamb
%gamma
```

Appendix (B): Wind Turbine Aerodynamics And Parameters

Starting from wind turbine dynamics equation:

$$J_T.\omega_T = T_A.T_L \tag{B.1}$$

Where (T_A) is the aerodynamic torque represented by :

$$T_A = \frac{1}{2}\rho ARCq(\lambda,\beta)u^2 \tag{B.2}$$

Assuming, ($T_A|_{OP} = T_L|_{OP}$), linearization of the equation (14) results in the following :

$$J_T \dot{\omega} = J_T \frac{\partial \dot{\omega}}{\partial u}\Big|_{OP} \Delta u + J_T \frac{\partial \dot{\omega}}{\partial \omega}\Big|_{OP} \Delta \omega + J_T \frac{\partial \dot{\omega}}{\partial \beta}\Big|_{OP} \Delta \beta \qquad \text{(B.3)}$$

simplifying the expression:

$$\dot{\omega} = \alpha \Delta u + \gamma \Delta \omega + \delta \Delta \beta \qquad \text{(B.4)}$$

Derivation of (α):

$$\alpha = \frac{\partial \dot{\omega}}{\partial u}\bigg|_{OP} = \frac{1}{J_T}\left[\frac{\partial T_A}{\partial u} - \frac{\partial T_L}{\partial u}\right]_{OP} = \frac{1}{J_T}\left[\frac{1}{2}\rho A R u^2 \frac{\partial C_q(\lambda,\beta)}{\partial u} - 0\right]_{OP} = \frac{1}{2J_T}\rho A R\left[u^2 \frac{\partial C_q(\lambda,\beta)}{\partial u}\right]_{OP} \qquad \text{(B.5)}$$

Table (1): Wind Turbine Characteristics [14]

Rated power (P_R)	20 kW
Radius (R)	5 m
Drive Train Inertia (J_T)	1270 kg.m^2
Gear ratio (n)	11.43
Operation Angular Speed (ω)	105 rpm
Cut- in Speed (u_c)	6.5 m.s^{-1}
Rated Wind Speed (u_R)	11 m.s^{-1}
Furling Speed (u_F)	23 m.s^{-1}
(β) Range	0 – 12 degree

Table (2): Aerodynamic Parameters for Grumman Windstream-33 Wind Turbine [6]

$\beta = 3^o$	$Cp = Cp_{max}$	0.42
	λ	7
	u	7.5 m.s^{-1}
$\beta = 9^o$	Cp	0.2
	$Cp = Cp_{max}$	0.28 at λ=5, u=11 m.s^{-1}
	λ	7
	u	7.5 m.s^{-1}
$\beta = 12^o$	Cp	0
	λ	7
	u	7.5 m.s^{-1}

Table (3): Wind Turbine Linearization Constants

α	$0.117\ s^{-2}$
δ	$-0.8582\ s^{-2}$
γ	$-0.0256\ s^{-1}$

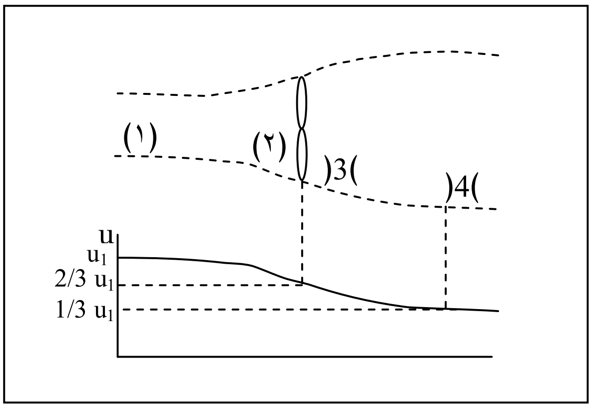

Fig. 1.Circular Tube of air Flowing Through A Wind Turbine

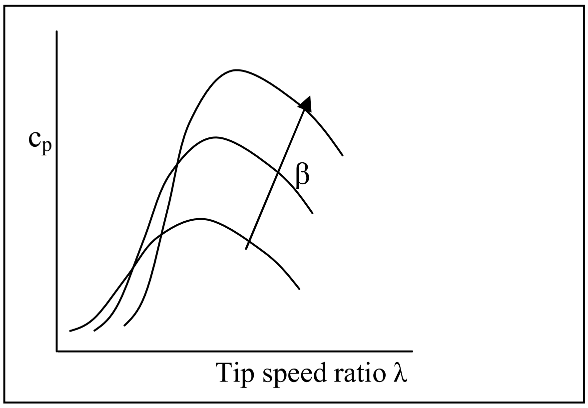

Fig. 2. General Trend of Cp vs λ and β

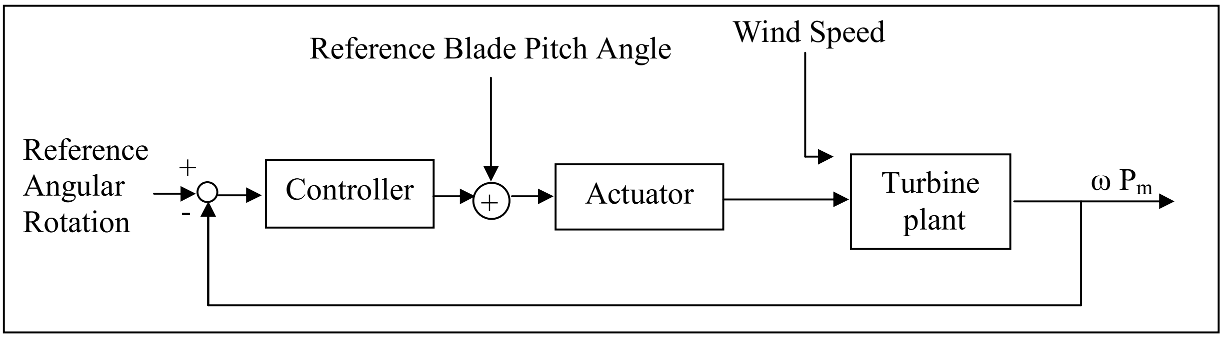

Fig. 3. Block Diagram of Wind Turbine system dynamic and Control

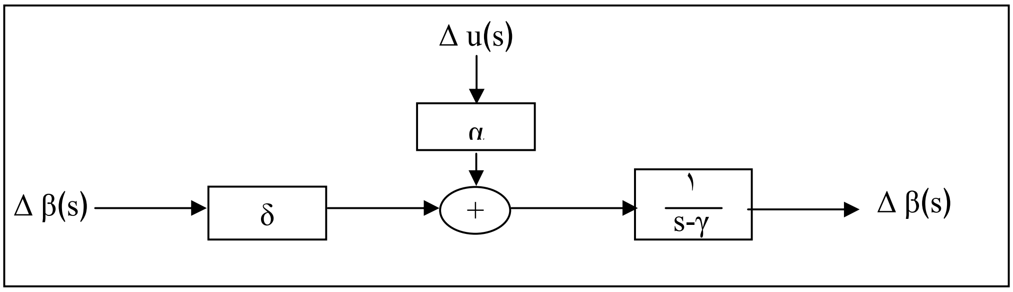

Fig. 4. Block Diagram of the linear Wind Turbine plant model and Control

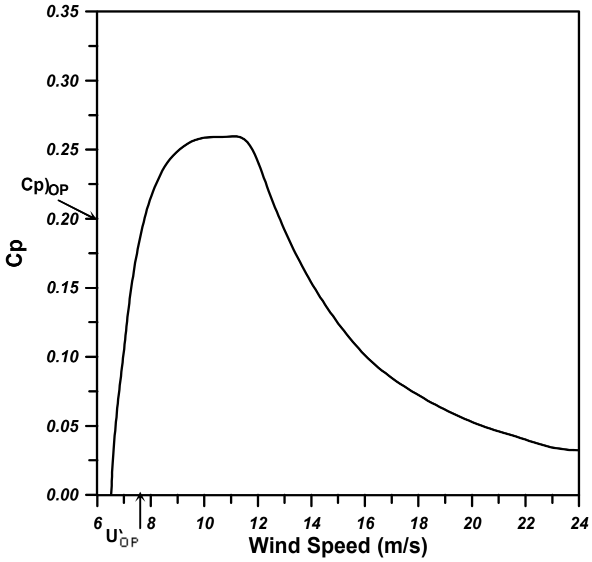

Fig. 5. Cp vs. U at fixed angel β of 9^O for Grumman Windstream

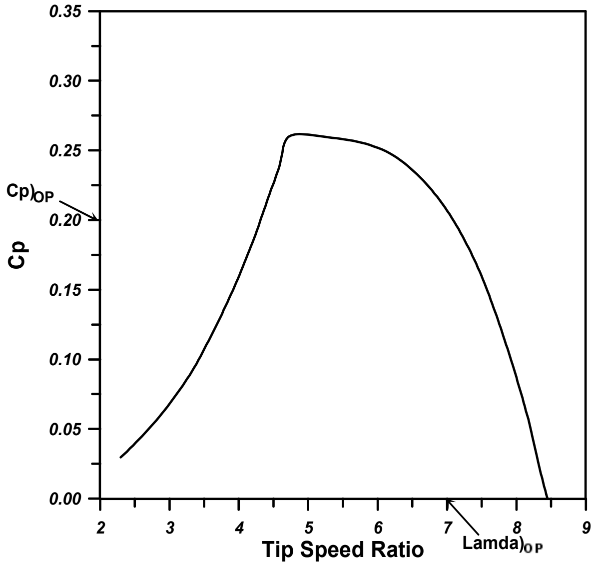

Fig. 6. Cp vs. λ at fixed angel β of 9^O for Grumman Windstream

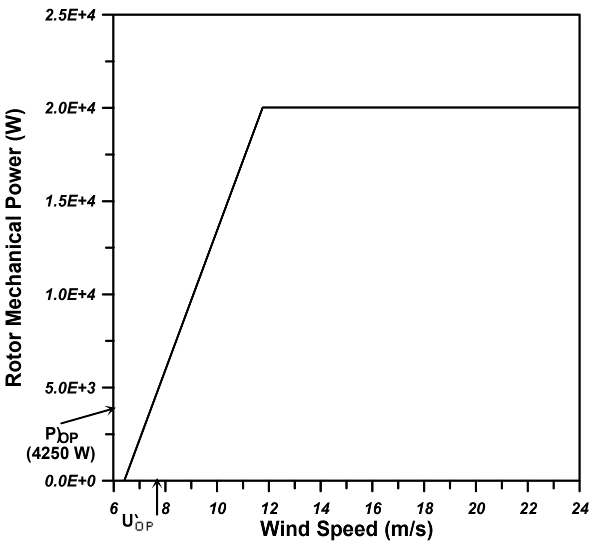

Fig. 7. P_m vs. U at fixed angel β of 9^O for Grumman Windstream-33

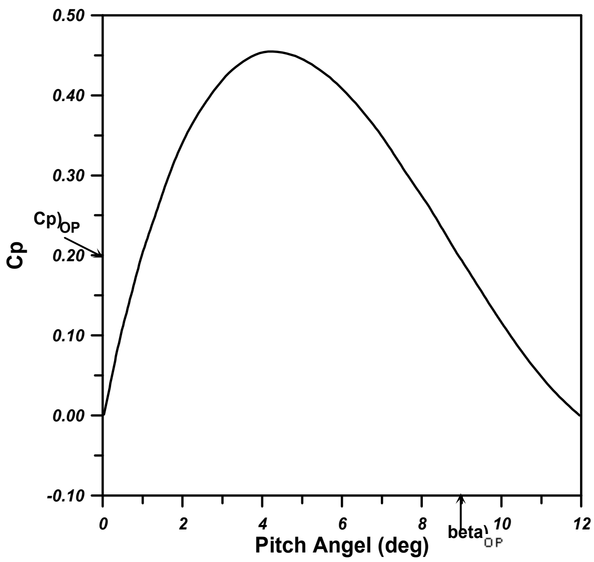

Fig. 8. Cp vs. U at fixed tip speed ratio of 7 for Grumman Windstream-33