

## TAIL PLANE DESIGN FOR SATISFYING LONGITUDINAL HANDLING QUALITIES

Dr. Farag M. Mohammed  
Mech. Eng. Dept. College of Engineering  
Anbar University.  
[drfaragmahel@yahoo.com](mailto:drfaragmahel@yahoo.com)

Dr. Yahya A. Faraj  
Mech. Eng. Dept. College of Engineering  
AL-Mustanesrya University.

### ABSTRACT

The Cooper-Harper rating of aircraft handling qualities has been adopted as a standard for measuring the performance of aircraft. In the present work, the tail plane design for satisfying longitudinal handling qualities has been investigated with different tail design for two flight conditions based on the Shomber and Gertsen method. Tail plane design is considered as the tail/wing area ratio. Parameters most affecting on the aircraft stability derivative is the tail/wing area ratio. The longitudinal handling qualities criteria were introduced in the mathematical contributions of stability derivative. This design technique has been applied to the Paris Jet; MS 760 Morane-Sualnier aircraft. The results show that when the tail/wing area ratio increases the aircraft stability derivative increases, the damping ratio and the natural frequency increases and the aircraft stability is improved. Three regions of flight conditions had been presented which are satisfactory, acceptable and unacceptable. The optimum tail/wing area ratio satisfying the longitudinal handling qualities and stability is  $(0.025 < S_t/S < 0.2)$ .

**KEY WORDS:** Longitudinal Handling, Stability, Tail Design.

### INTRODUCTION

The link between stick force per (g) and maneuver margin and hence the longitudinal short period stability of an aircraft makes it clear that the handling characteristics must still dominate the design and that what is required is a new means of presenting satisfactory handling characteristics. The tail plane to wing design is often associated with either lifting the nose at the required speed during the take off or just before touch down during landing. In both cases with a forward center of gravity position a large down load on the tail plane is required at a low equivalent airspeed at a time when ground effect can reduce the downwash at the tail to a near zero value.

Handling qualities may be defined as those dynamic and static properties of a vehicle that permit the pilot to fully exploit its performance in a variety of missions and roles. Traditionally, handling quality is measured using the Cooper-Harper rating (CHR) and done subjectively by the human pilot [1].

Timothy H. Cox and Dante W. Jackson [2] had presented the most flying qualities developed from data in the subsonic flight regime and discovered a good correlation between some of the classical handling qualities parameters, such as the control anticipation parameter as a function of damping. Timothy H. Cox and Alisa Marshall [3] investigated the longitudinal handling qualities of the Tu-144LL Airplane. Here, four flights had been conducted using the Tu-144 airplane with the dedicated objective of collecting quantitative data and qualitative pilot comment. These data were compared with the longitudinal flying qualities criteria: Neal-Smith, short period damping, time delay, control anticipation parameter, phase delay, pitch bandwidth as a function of time delay and flight path as a function of pitch bandwidth. The data showed that

the approach and landing requirements appear to be applicable to the precision flight control required for up and away flight.

Ilhan Tuzcu [4] studied the dynamic and control of flexible aircraft and he integrated in a single mathematical formulation the disciplines pertinent to the flight of flexible aircraft. The unified formulation is based on fundamental principles and incorporates in a natural manner both rigid body motions of the aircraft as whole and elastic deformations of the flexible components, as well as the aerodynamic, propulsion, gravity and control forces. The aircraft motion was described in terms of three translations and three rotations of a reference frame attached to the undeformed fuselage and acting as aircraft body axes, and elastic displacements of each of the flexible components relative to corresponding body axes.

Phillips Charles [5] had estimated the aerodynamic and handling qualities of the C-130 a modified with wing tip tanks. This work presented the background, flight testing, and resulting change in the aerodynamic and handling qualities of a C-130A Hercules modified with wing tip tanks, and showed that the lift benefits of these uniquely designed tip tanks for the C-130A cargo transport proved that by capitalizing on the benefits of a combination tip tank and end plate design it is possible to generate increased lift without adversely affecting the stability and dynamic parameters of the aircraft.

Methods of presenting of the Cooper-Harper rating (CHR) of handling characteristics were depending on the pilot evaluation. The present work is introducing a method based on Shomber and Gertsen [6] method of assessing longitudinal handling characteristics. It is assumed that the first estimates of weights and measurements are known, including the wing, tailplane position and aspect ratio.

### EQUATIONS OF LONGITUDINAL MOTION.

The characteristic modes of stick fixed longitudinal motion for really all airplanes are two oscillations, one of long period with poor damping (phugoid mode) and the other of short period with heavy damping referred to short period mode. The linearised, Laplace transformed longitudinal short period mode Eqn. of motion of aircraft are given by [7]:

$$\begin{aligned} \left( \frac{mU}{Sq} s - C_{z\alpha} \right) \alpha'(s) - \frac{mU}{Sq} s \theta(s) &= C_{z\delta}(s) \delta(s) \\ \left( \frac{-c}{2U} C_{m\dot{\alpha}} s - C_{m\alpha} \right) \alpha'(s) + \left( \frac{I_y}{Sq} s^2 - \frac{c}{2U} C_{m\dot{\theta}} s \right) \theta(s) &= C_{m\delta} \delta(s) \end{aligned} \quad (1)$$

The short period mode transfer function of an aircraft is  $\frac{\alpha'(s)}{\delta(s)}$  or  $\frac{\theta(s)}{\delta(s)}$  as shown in Fig.

(1). The characteristic Eqn. of the transfer functions is given as:

$$\begin{vmatrix} \frac{mU}{Sq} s - C_{z\alpha} & -\frac{mU}{Sq} s \\ \frac{-c}{2U} C_{m\dot{\alpha}} s - C_{m\alpha} & \frac{I_y}{Sq} s^2 - \frac{c}{2U} C_{m\dot{\theta}} s \end{vmatrix} = 0 \quad (2)$$

The expansion of this expression can be written in the form of:

$$s(As^2 + Bs + C) = 0 \tag{3}$$

Where

$$A = \left( \frac{I_y}{Sq} \right) \left( \frac{mU}{Sq} \right)$$

$$B = \left( \frac{-c}{2U} C_{mq} \right) \left( \frac{mU}{Sq} \right) - \frac{I_y}{Sq} C_{Z\alpha} - \left( \frac{c}{2U} C_{m\dot{\alpha}} \right) \left( \frac{mU}{Sq} \right)$$

$$C = \frac{c}{2U} C_{mq} C_{Z\alpha} - \frac{mU}{Sq} C_{m\alpha}$$

If Eq. (3) is divided by (A) and written in the standard form of quadratic  $\zeta$  and  $\omega_n$ , it results in:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Where

$$\omega_n = \frac{U\rho S c}{2} \left( \frac{\frac{1}{2} C_{m\dot{\theta}} C_{Z\alpha} - \frac{2m}{\rho S c} C_{m\alpha}}{I_y m} \right)^{\frac{1}{2}} \tag{4}$$

$$\zeta = -\frac{1}{4} \left( C_{m\dot{\theta}} + C_{m\dot{\alpha}} + \frac{2I_y}{m c^2} C_{Z\alpha} \right) \left( \frac{m c^2}{I_y \left( \frac{C_{m\dot{\theta}} C_{Z\alpha}}{2} - \frac{2m C_{m\alpha}}{\rho S c} \right)} \right)^{\frac{1}{2}}$$

In literature the researchers had found that the most parameters affecting on the damping ratio and natural frequency of short period are the pitching moment coefficient due to change angle of attack ( $C_{m\alpha}$ ) which is used to determine the static longitudinal stability of aircraft, and the pitching moment coefficient due to a pitch rate ( $C_{m\dot{\theta}}$ ). Therefore in this work the study will be concerned with the variation of these two parameters. Where:

$$C_{m\alpha} = \frac{\partial C_m}{\partial \alpha} = \left( \frac{dC_m}{dC_L} \right) \left( \frac{dC_L}{d\alpha} \right)$$

$$C_{m\dot{\theta}} = \frac{\partial C_m}{\partial \dot{\theta}} = \left( \frac{dC_m}{dC_L} \right) \left( \frac{dC_L}{d\dot{\theta}} \right) \tag{5}$$

From Figs. (2 and 3) the pitching moment coefficient about the aircraft center of gravity can be written as:

$$C_{mcg} = C_N \frac{x_a}{c} + C_A \frac{z_a}{c} + C_{mac} + C_{mfus} - C_{Nt} \frac{S_t}{S} \frac{\ell_t}{c} \eta_t \quad (6)$$

Where:

$$\begin{aligned} C_N &= C_L \cos(\alpha - i_w) + C_D \sin(\alpha - i_w) \\ C_A &= C_D \cos(\alpha - i_w) - C_L \sin(\alpha - i_w) \\ C_{Nt} &= \left( \frac{dC_N}{d\alpha} \right)_t \alpha_t \\ \alpha_t &= \alpha - \varepsilon + i_t - i_w \\ \varepsilon &= \frac{d\varepsilon}{d\alpha} (\alpha_w - \frac{d\alpha}{dt} \Delta t) \\ \Delta\alpha &= \dot{\theta} \frac{\ell_t}{U} \\ \Delta t &= \frac{\ell_t}{U} \end{aligned} \quad (7)$$

The stability derivatives will be shown to be a function of the lift coefficient, and the slope of the curve of pitching moment coefficient plotted against lift coefficient. The slope can be obtained analytically by differentiating Eq. (6) with respect to  $C_L$  and assuming that the drag coefficient is small relative to lift coefficient in all cases:

$$\frac{dC_{mcg}}{dC_L} = \underbrace{\frac{dC_N}{dC_L} \frac{x_a}{c} + \frac{dC_A}{dC_L} \frac{z_a}{c}}_{\text{wing contribution}} + \underbrace{\frac{dC_{mac}}{dC_L}}_{\text{fuselage contribution}} + \underbrace{\frac{dC_{mfus}}{dC_L} - \frac{dC_{Nt}}{dC_L} \frac{S_t}{S} \frac{\ell_t}{c} \eta_t}_{\text{tail contribution}} \quad (8)$$

Where  $(X_a = \frac{x_a}{c} = X_{cg} - X_{ac})$  is the distance between the aerodynamic center and the aircraft center of gravity in length of the mean chord. In the present work, it is assumed that the first estimates of weight and measurement are known and the investigation is done on the variation of position and size of wing and tail plane. To introduce the contributions of wing and tail Eq. (7) is substituted in Eq. (8), then the stability derivative Eq. becomes:

$$\begin{aligned} C_{m\alpha} &= X_a a - \left( \frac{dC_m}{dC_L} \right)_{fus.} a - a_t \frac{S_t}{S} \frac{\ell_t}{c} \left( 1 - \frac{d\varepsilon}{d\alpha} \right) \eta_t \\ C_{m\dot{\theta}} &= -a_t \frac{S_t}{S} \left( \frac{\ell_t}{c} \right)^2 \eta_t \end{aligned} \quad (9)$$

## LONGITUDINAL HANDLING CRITERIA

The Cooper-Harper rating of aircraft handling qualities has been adopted as a standard for measuring the performance of aircraft. Aircraft performance, ability to control the aircraft,

and the degree of pilot compensation needed are three major key factors used in deciding the aircraft handling qualities in the Cooper-Harper rating. The automatic estimate of the system-level handling quality provides valuable up-to-date information for diagnostics and vehicle health management. Analyzing the performance of a controller requires a set of concise design requirements and performance criteria. In the case of control systems for a piloted aircraft, generally applicable quantitative design criteria are difficult to obtain. The reason for this is that the ultimate evaluation of a human-operated control system is necessarily subjective and, with aircraft, the pilot evaluates the aircraft in different ways depending on the type of the aircraft and the phase of flight. In most aerospace applications (e.g., for flight control systems), performance assessment is carried out in terms of handling qualities.

In this work, a method based on Shomber and Gertsen [6] method of assessing longitudinal handling characteristics is introduced. Fig. (4) in reference [6] represents the relationship between the aircraft lift per unit moment ( $L_\alpha/\omega$ ) and the longitudinal short period damping ratio ( $\zeta$ ) for values of normal acceleration in (g) units per unit incidence ( $n_\alpha$ ) less than or equal fifteen, where:

$$L_\alpha = \frac{L}{Vm} = \frac{\rho SV a}{2m} \tag{10}$$

$$\frac{L_\alpha}{\omega_n} = \frac{\rho SV a}{2m\omega_n}$$

Where ( $\omega_n$ ) is the undamped natural frequency of that mode. Fig. (5) in reference [6] represents the relationship between the ( $n_\alpha/\omega$ ) and ( $\zeta$ ), and is appropriate for flight condition in which ( $n_\alpha$ ) exceeds fifteen, where:

$$n_\alpha = \frac{L_\alpha V}{g} = \frac{\rho SV^2 a}{2mg} \tag{11}$$

$$\frac{n_\alpha}{\omega_n} = \frac{L_\alpha V}{g\omega_n} = \frac{\rho SV^2 a}{2mg\omega_n}$$

These two Figures show the boundary between the satisfactory (region-1), acceptable (region-2), and not acceptable (region-3) handling characteristics.

The correct wing position is found by loading the aircraft appropriate to the aft center of gravity position and then moving the wing group components forward relative to the fuselage group until the distance between the aerodynamic center and the center of gravity corresponds to the maximum limiting value of ( $X_a$ ) appropriate to the tailplane selected. For an equilibrium in a given flight condition, to estimate the distance between the center of gravity and the aerodynamic center, the Eqn. in pitch which must sum up to ( $C_{mcg}=0$  and  $\frac{dC_{mcg}}{dC_L}=0$ ), get the neutral point. Introducing the longitudinal handling qualities from Fig. (4) of Shomber and Gertsen in the ( $X_a$ ) relation results in:

$$X_a = \frac{\frac{a_t S_t}{a S} \left[ 1 + \frac{2m}{a} \left( 1 - \frac{d\varepsilon}{d\alpha} \right) \right] - \left[ \frac{VK}{g \ell_t \frac{n_a}{\omega_n}} \right]}{\frac{2mc}{a \ell_t} \left[ 1 + \frac{a_t S_t}{a S} \left( 1 - \frac{d\varepsilon}{d\alpha} \right) \right]} \quad (12)$$

The tail plane design ( $S_t/S$ ) can be expressed also by introducing longitudinal handling qualities from Figs. (4 and 5) of Shomber and Gertsen in stability derivative as:

$$\frac{S_t}{S} = - \frac{\frac{a}{a_t} \left( \frac{K}{\ell_t} \right)^2}{1 + \frac{d\varepsilon}{d\alpha}} \left[ \frac{2\zeta}{L_\alpha / \omega_n} - 1 \right] \quad (13)$$

### RESULTS AND DISCUSSION

The design technique has been applied to the Paris Jet, MS 760 Morane-Sualnier aircraft. The prototype MS.760A Paris shown in Fig. (6) had a low wing and was powered by two Marbré 400kg engines, mounted side by side in the fuselage. It was recognizable by its T-shaped vertical stabilizer and by its retractable tricycle landing gear. The aircraft had four seats, two in the front and two in the back. The airplane specifications are listed in Table (1).

Table (1)  
Paris Jet, MS 760 Morane-Sualnier aircraft specifications [8].

Length	10.42 m
Wing span	10.15 m
Wing area	18 m <sup>2</sup>
Height (to top tail fin)	2.6 m
Stabilizer span	3.35 m
Maximum take off weight	3924 kg
Maximum landing weight	3157 kg
Maximum speed	695 km/hr
Maximum ceiling	25000 ft

Fig. (7) shows the behavior of the Pitching moment due to angle of attack change (longitudinal static stability  $C_{m\alpha}$ ) with the variation of tail plane design with different airplane center of gravity positions. At neutral point when the center of gravity move most aft and coincide with the aerodynamic center ( $X_a=0$ ), the behavior shows that the longitudinal static stability increases in absolute value with the tail/wing area ratio increases and the airplane become more stable rather than neutral stable. Also the two other behavior of the longitudinal stability at which the airplane center of gravity most forward show stability derivative increase with the area ratio increase, because when the area ratio of tail to wing increases an opposite pitching moment produced by the tail lift and drag about the airplane center of gravity is also increased.

Fig. (8) shows the tail plane design effect on the stability derivatives. When the tail to wing area ratio increases the two stability derivatives increase in negative value because this ratio is the most dominant parameter in Eq. (9). Fig. (9) presents the variation of natural frequency with tail design. With the tail to wing area ratio increase the natural frequency increases, the natural frequency and damping ratio were depending on the stability derivatives and these derivatives are most dependent on the area ratio as mentioned before in Eq. (4). Also Fig. (10) presents the variation of damping ratio with tail design for two flight conditions at CHR of 3.5 and 6.5 values, and have more damped aircraft with area ratio increase. Three regions were pointed as referred to satisfactory, acceptable and unacceptable regions of flight according to the resulting damping ratio. The acceptable region is limited by  $(0.025 < S_t/S \leq 0.22)$ .

Eq. (13) is used to find the corresponding value of  $(S_t/S)$ . Thus a pair of values  $(L_\alpha/\omega$  and  $\zeta)$  from Fig. (1) with different CHR and reference [6], can be used to find a pair of  $(S_t/S$  and  $L_\alpha/\omega_n)$  corresponding to the constant pilot rating as shown in Fig. (11). Also three regions were pointed as referred to the satisfactory, acceptable and unacceptable regions of flight. The acceptable region is limited by  $(0.02 < S_t/S < 0.28)$ .

Eq. (12) is used to find the corresponding value of  $(X_a)$ . Thus a pair of values  $(n_\alpha/\omega_n$  and  $\zeta)$  from Fig. (2) reference [6] at different CHR can be used to find a pair of  $(X_a$  and  $S_t/S)$  corresponding to the constant pilot rating as shown in Fig. (12). Also three regions were pointed as referred to the satisfactory, acceptable and unacceptable regions of flight. The acceptable region is limited by  $(0.025 < S_t/S < 0.2)$ , because out of this region, the airplane center of gravity becomes aft of the aerodynamic center and become unstable.

## CONCLUSIONS

The tail plane design satisfying the airplane longitudinal handling qualities was studied with variation of tail to wing area ratio based on Shomber and Gertsen method. The following conclusions are drawn:

1. The longitudinal static stability increased in absolute value with the tail to wing area ratio increase and resulted in a more stable airplane.
2. The natural frequency and damping ratio increased with the tail/wing area ratio increased and resulted in a more damped mode.
3. The stability is satisfying at neutral point with selection of suitable area ratio.
4. The optimum design tail/wing area ratio is happened at  $(0.025 < S_t/S < 0.2)$  for satisfying the longitudinal handling qualities.

## REFERENCES

- [1] Lionel D. Alford and Robert C. Knarr, "General flight test theory applied to aircraft modifications", Acquisition Review Quarterly, pp. 157-167, Spring 1999.
- [2] Timothy H. Cox and Dante W. Jackson, "Evaluation of high-speed civil transport handling qualities criteria with supersonic flight data", NASA Technical Memorandum 4791, April-1997.
- [3] Timothy H. Cox and Alisa Marshall, "Longitudinal handling qualities of the Tu-144LL airplane and comparisons with other large supersonic aircraft", NASA-TM-2000-209020, May-2000.
- [4] Ilhan T. "Dynamics and control of flexible aircraft", PhD thesis, Virginia Polytechnic Institute and State University, December-2001.

- [5] Phillips C. "Analysis and parameter estimation of the aerodynamic and handling qualities of the C-130 a modified with wing tip tanks", MSc thesis, North Carolina State University, 2004.
- [6] Shomber, H. and Gertsen, W. "Longitudinal handling qualities criteria: an evaluation", Journal of aircraft vol. 4 No. 4, July/August-1967.
- [7] John, H. B. "Automatic control of aircraft and missiles", John Wiley and sons, Inc., New York, 2000.
- [8] Flight research Inc. <http://www.flightresearch.com/aircraft/Paris.htm>
- [9] Duncan, W. J. "Control and stability of aircraft", Cambridge University Press, 1952.
- [10] Courtland D. P. and Robert E. H. "Airplane performance stability and control", John Wiley and sons, Inc., New York, 1967.

### SYMBOLS

a	Lift curve slope.
c	Airfoil chord (m).
C	Coefficient.
D	Drag force (N).
i	Incidence angle (deg).
$I_y$	Moment of inertia ( $m^4$ ).
$\ell$	Length (m)
L	Lift force (N)
$L_\alpha$	The aircraft lift per unit moment (1/sec).
m	Aircraft mass (kg).
q	Dynamic pressure ( $N/m^2$ ).
S	Wing area ( $m^2$ ).
t	Time (sec.)
U	Air speed (m/sec).
$X_a$	Distance between aerodynamic center and center of gravity(m)
$\alpha$	Angle of attack (deg).
$\delta$	Elevator angle (deg).
$\varepsilon$	Dawn wash angle (deg).
$\rho$	Air density ( $kg/m^3$ ).
$\theta$	Attitude angle (deg).
$\omega_n$	natural frequency (rad/sec).
$\eta$	Efficiency.
$\zeta$	Damping ratio.

### Subscript:

A	Axial.
ac	Aerodynamic center.
cg	Center of gravity.
L	Lift.
m	Moment.
$m_\alpha$	Dawn wash lag on moment.



- $m\alpha$  Moment due to angle of attack.
- $m\delta$  Moment due to angle of elevator.
- $m\dot{\theta}$  Moment due damping in pitch.
- N Normal.
- t Tail.
- $z\delta$  Normal force due to elevator angle.

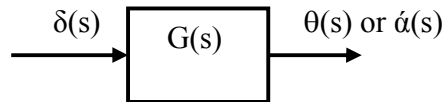


Figure (1): open loop block diagram for the aircraft.

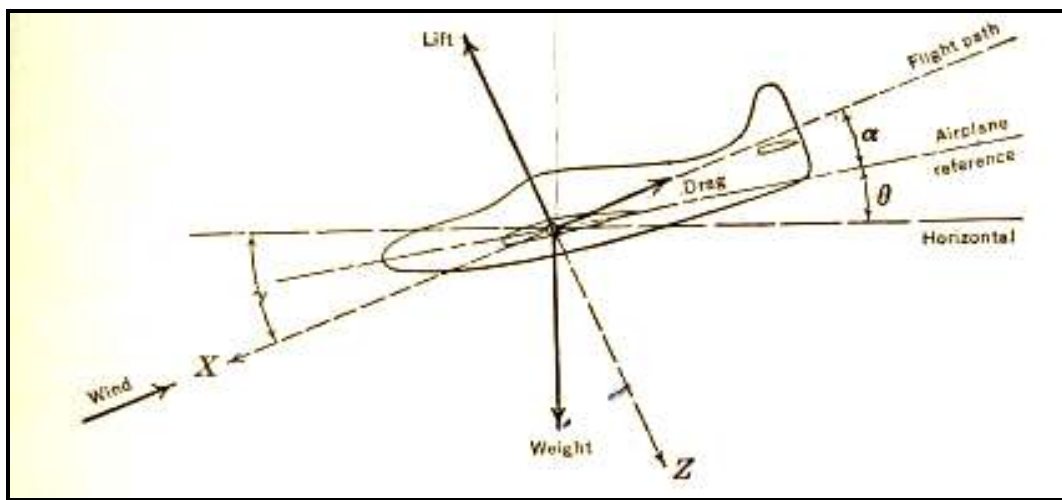


Figure (2): Aircraft attitude [9].

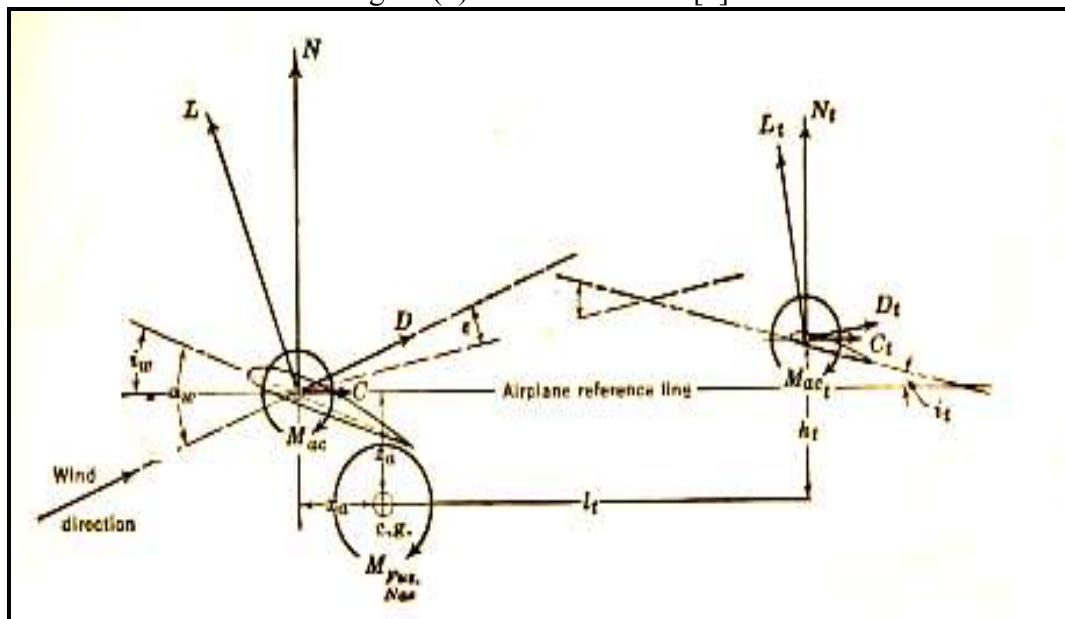


Figure (3): Forces and moments in plane of symmetry of aircraft [10].

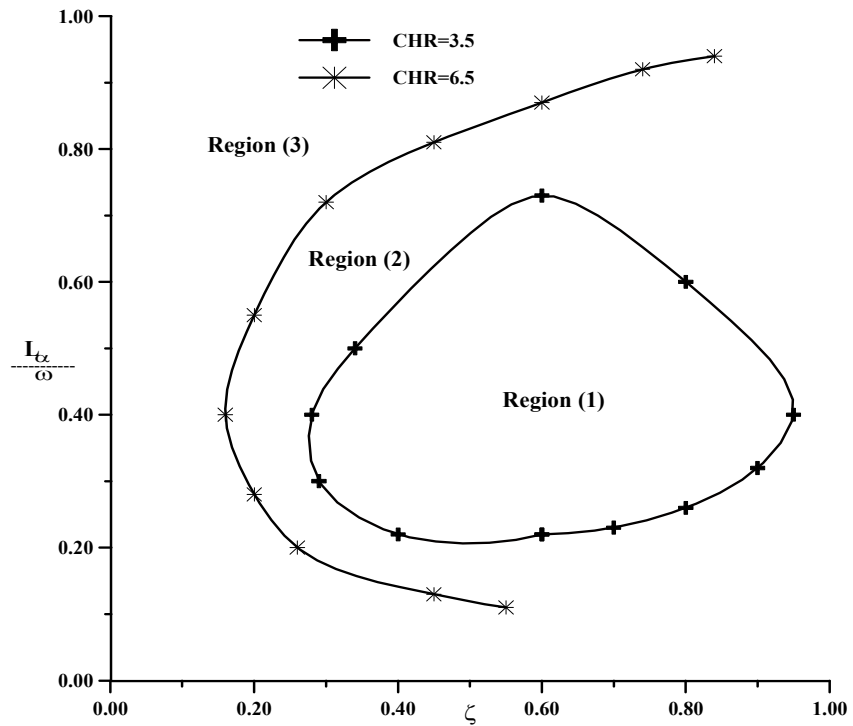


Figure (4): Longitudinal short period criteria for  $(n_\alpha < 15)$  [6].

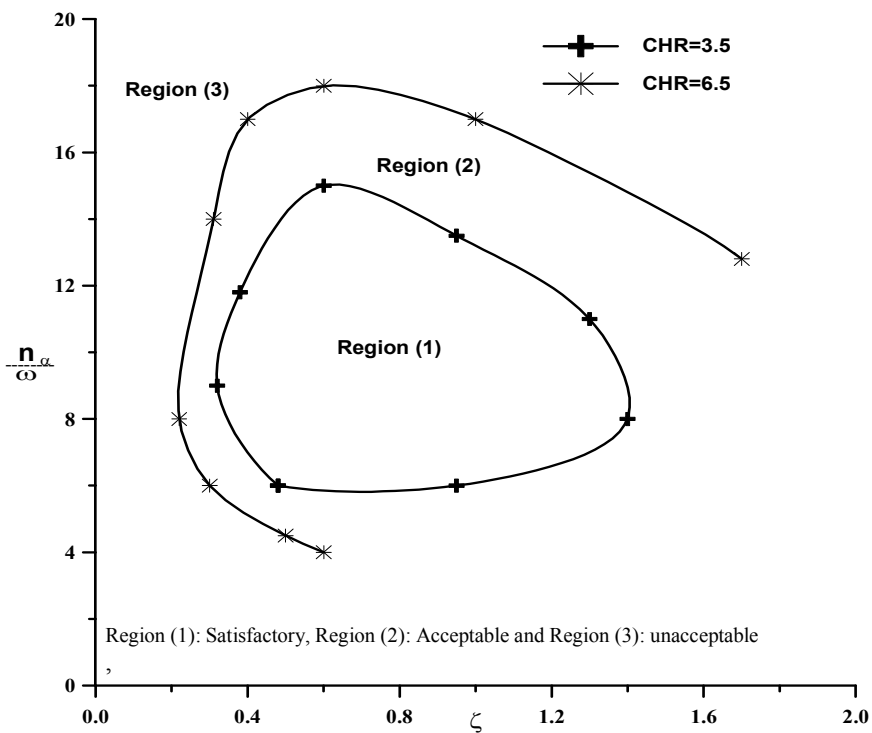


Figure (5): Longitudinal short period criteria for  $(n_\alpha > 15)$  [6]

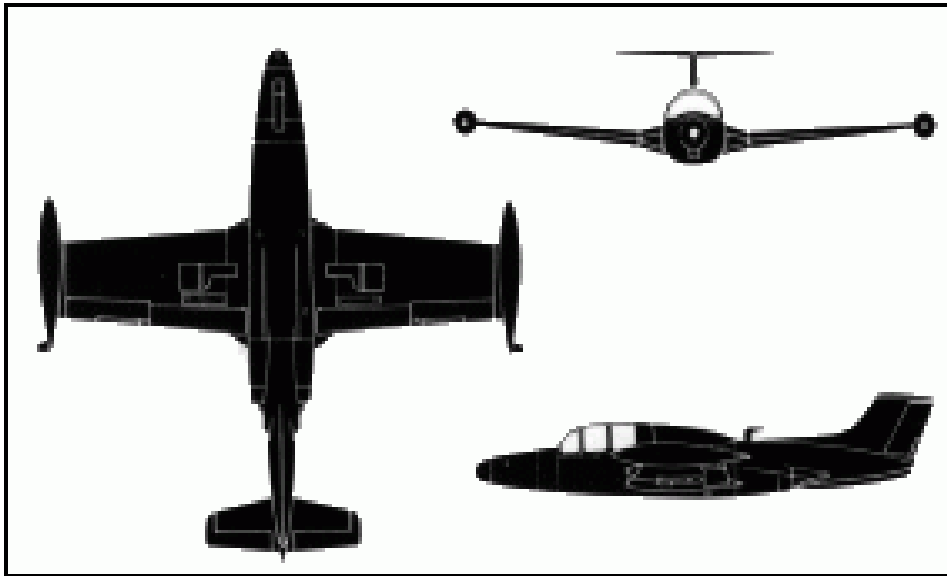


Figure (6): 3-view of Paris Jet, MS 760 Morane-Sualnier airplane [8].

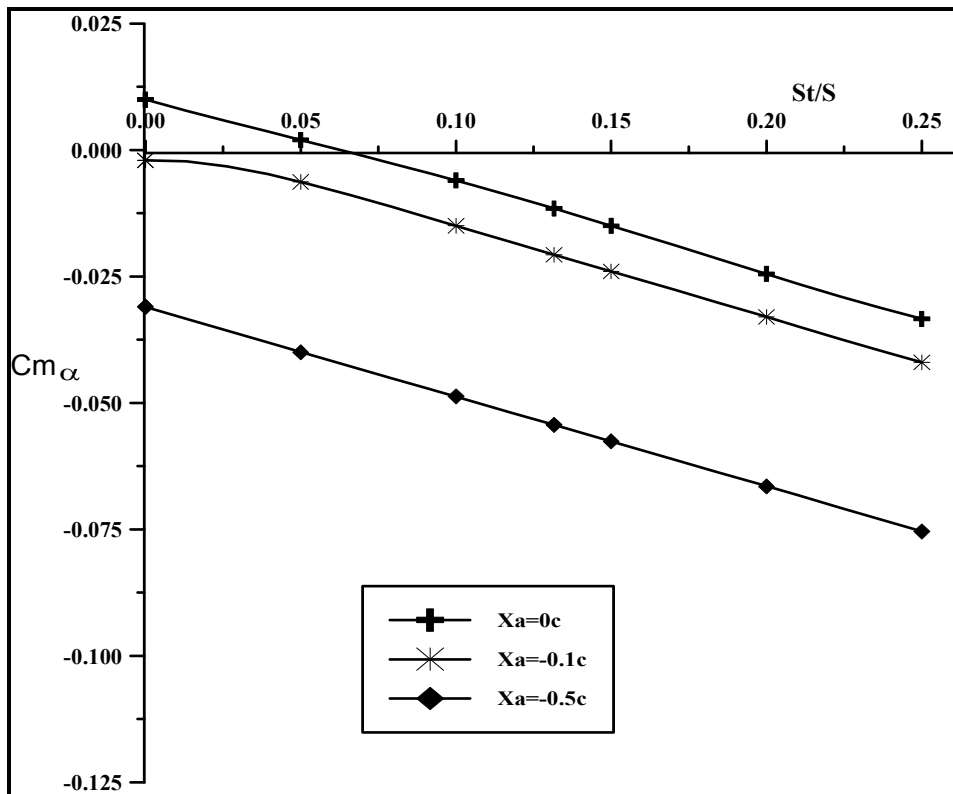


Figure (7): Effect of tail plane design on pitching moment due to change in angle of attack.

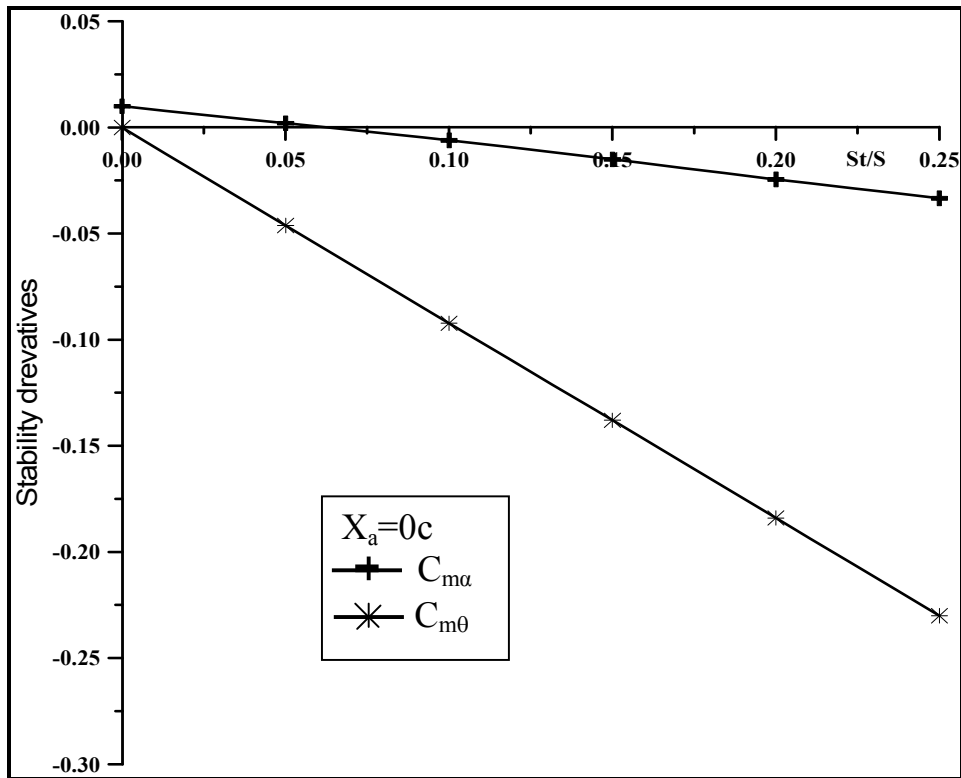


Figure (8): Effect of tail plane design on stability derivatives.

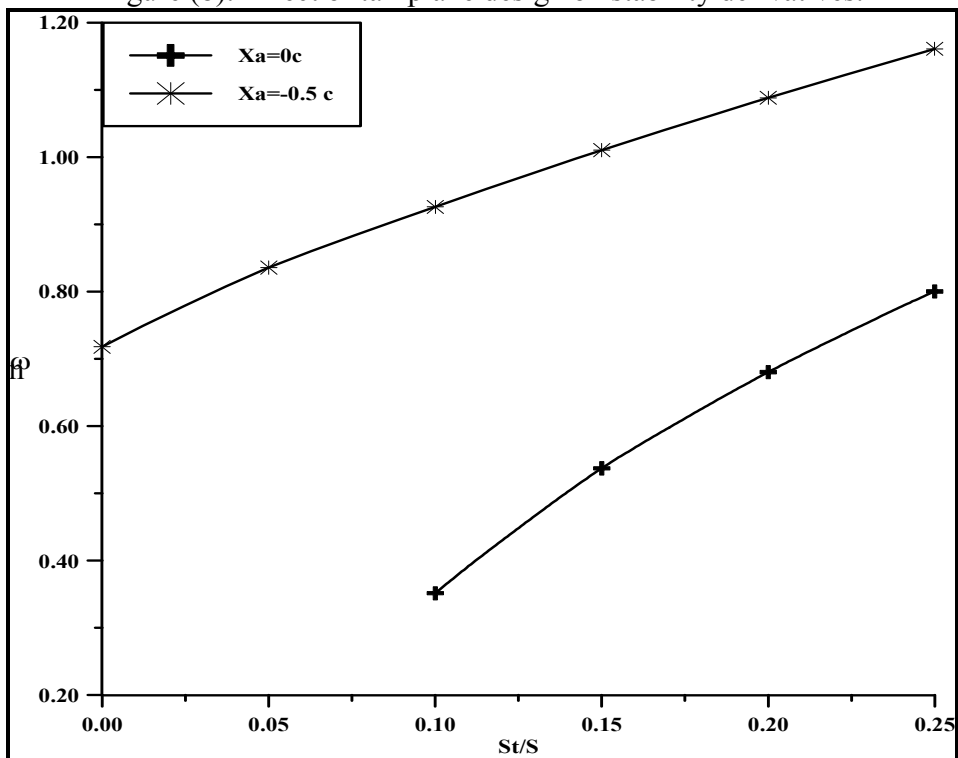


Figure (9): Effect of tail plane design on the aircraft natural frequency.

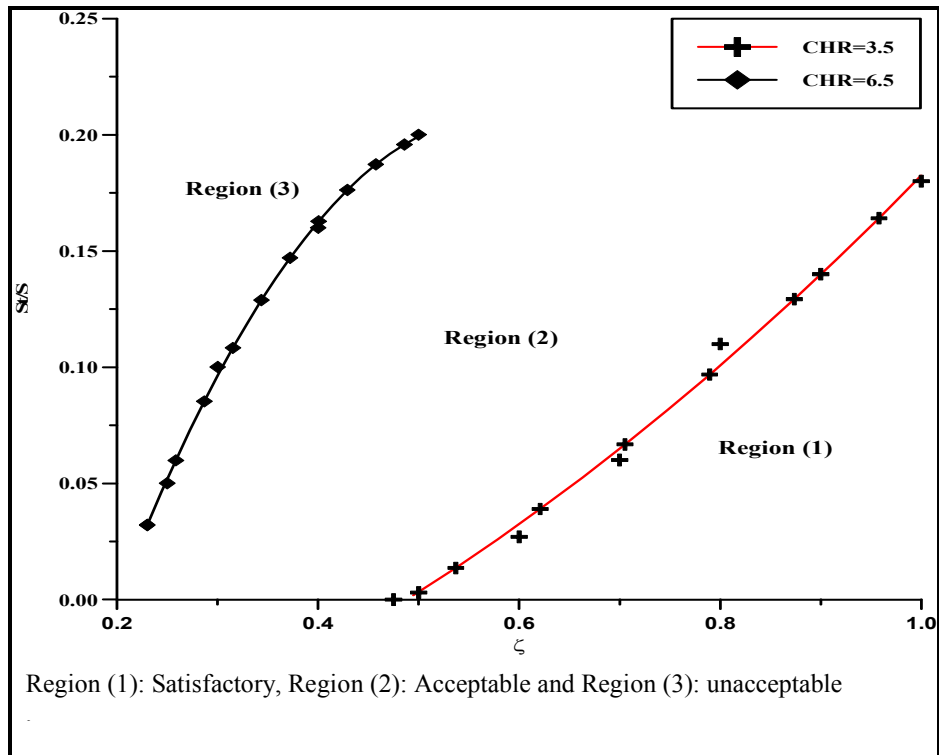


Figure (10): Effect of tail plane design on the aircraft damping ratio.

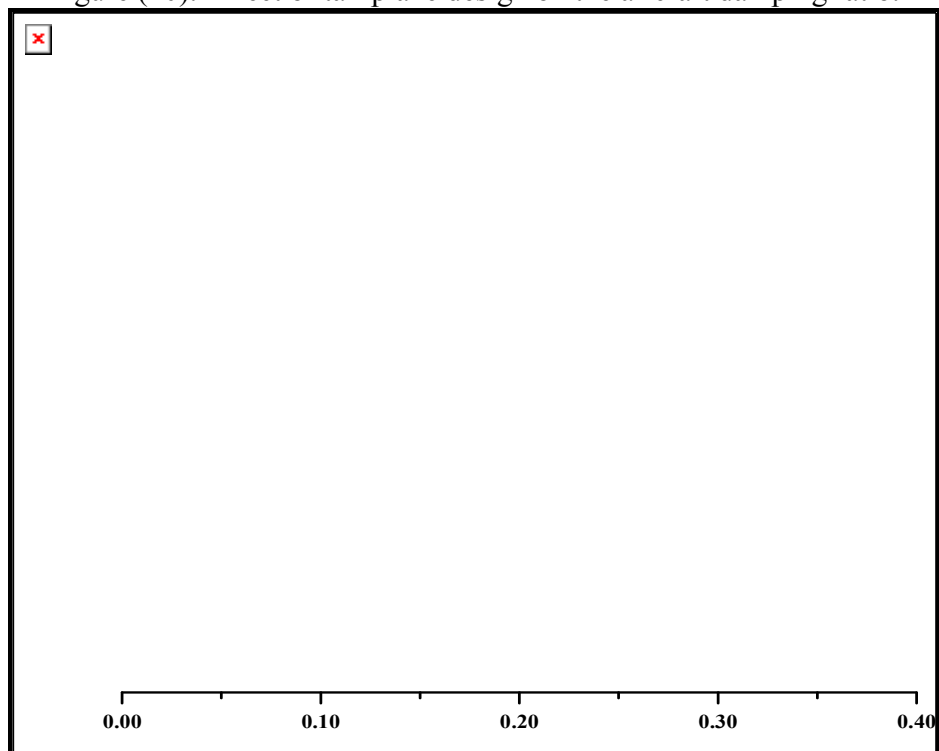


Figure. (11): Effect of tail plane design on the longitudinal short period criteria.

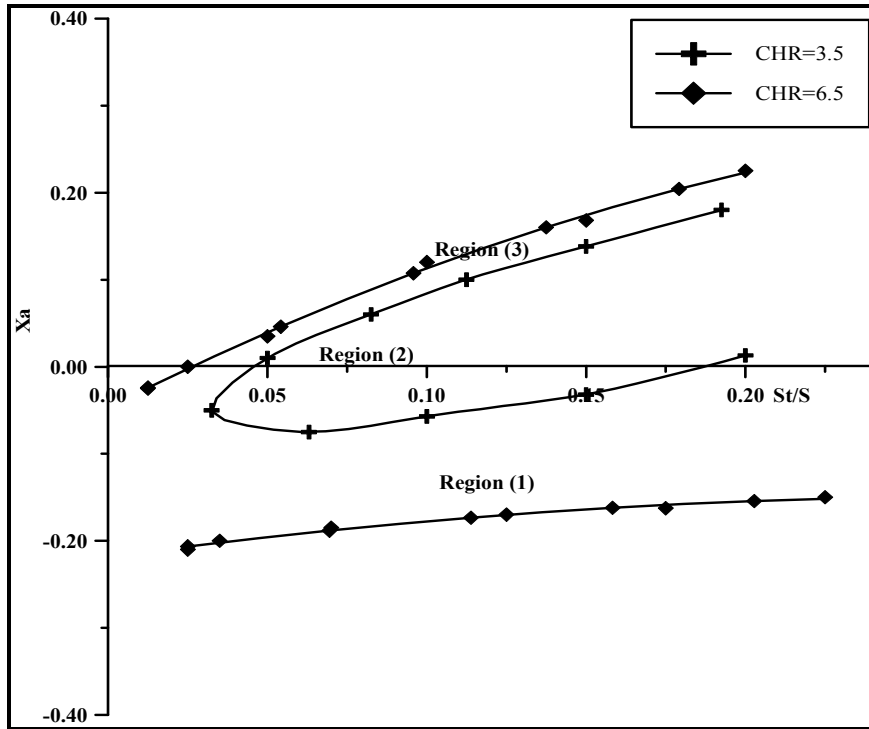


Figure (12): Effect of tail plane design on the airplane center of gravity travel.

## تصميم ذيل الطائرة لتلبية معالجة الخصائص الطولية

الدكتور يحيى عبدالله فرج  
الجامعة المستنصرية-كلية الهندسة

الدكتور فرج محل محمد  
جامعة الانبار-كلية الهندسة

### الخلاصة

يعتبر معدل كوبر-هاربر لمعالجة خصائص الطائرة هو المصطلح القياسي لحساب اداء الطائرة. في هذا البحث تمت دراسة تصميم ذيل الطائرة لتلبية معالجة الخصائص الطولية عند تصاميم مختلفة وحالتى طيران بالاعتماد على طريقة شومبر وجيتسن. ان تصميم الذيل اخذ كنسبة مساحة الذيل الى مساحة الجناح. اكثر المتغيرات تأثيرا على مشتقات الاستقرار الطولية هي نسبة مساحة الذيل الى مساحة جناح الطائرة. تم ادخال متغيرات معالجة الخصائص الطولية في المعادلات الرياضية للاستقرار الطولية. وتم تطبيق هذه التقنية التصميمية على الطائرة (Paris Jet; MS 760 Morane-Sualnier aircraft). اظهرت النتائج ان زيادة نسبة مساحة الذيل الى مساحة الجناح تؤدي الى زيادة مشتقات الاستقرار الطولية وكذلك زيادة نسبة التخميد والتردد الطبيعي والذي يؤدي الى تحسين استقرار الطائرة وسرعة الاستجابة والمعالجة. تم توضيح ثلاثة مناطق تكون فيها المعالجة الطولية متحققة؛ مقبولة وغير مقبولة. وتبين ان النسبة المثلى التي تحقق المعالجة والاستقرار الطولية المطلوبة هي  $(0.025 < S_t/S < 0.2)$ .