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# Dynamic Response of a Cracked Composite Beam subjected to moving Load

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**ABSTRACT**

The forced deflections of simply supported cracked composite beams are investigated when subjected to moving dynamic load. The crack is modeled as rotational spring and used in the formulation of the composite beam with a moving load in sinusoid wave. The numerical solution is satisfactory compared to the experimental results. The effects of crack depth and crack positions at different load speed are studied. The results show that the forced deflection increased with increasing the speed ratio and crack depth.



## 1. Introduction

The dynamic response of composite structures under moving load is one of the most interesting topics for structural researchers. L.Santosh sreekanth and M. Kumaraswamy [1] investigated a cracked composite beams of graphite fiber reinforced polyamide and E-glass polymer using ANSYS for the purpose of saving of time and cost in modelling. Andrzej katunin [2] studied the detection of crack in composite beams using of fractional B-spline wavelets. The data from finite element method was used for analysis of cracked composite beam. Sadettin Orhan, Jr. [3] presented a v-shaped crack model to study the influence of defect geometery on the natural frequencies and modeshapes of composite beams. Arjus S. Menon, Jr. [4] proposed a finite element model of the cantilever cracked composite beam with a ransverse one edge non propagating open crack. The depth of the crack, change in the natural frequencies as function of the angle of the fibers has been investigated. Andrzej Katunin and *Piotr Przystalka* [5] solved a problem of detectability of delamination in discrete wavelet transform and fractional B-spline wavelets method. The proposed method allowed for detection of delamination area in composite beams. E.Bahmyari, Jr. [6] studied the dynamic response of the inclined composite beams under distributed mass loading. The first order shear deformation and classical theories has been used in Finite Element Modeling. A good agreement was obtained in comparison with previous works. Arshad Mehmood, Jr. [7] investigated the dynamic response of the beam structure, frame and spring attached frames under moving load. Euler beam theory was used in the finite element method and Newmark integration for forced vibration analysis. S.R.Mohebpour, Jr. [8] developed an algorithm to analyze orthotropic unsymmetrical composite laminated beams subjected to moving mass load with a constant speed or acceleration. The presented computer code was more efficient than the other finite element analysis codes. Vahid sarvestan, Jr. [9] presented vibration response of a cracked beam under a moving load using spectral finite element method. The Fourier transform was used to transform moving load to spectral domain. The developed SEFM was use to analyse a viscoelastic Euler-Bernoulli beam. Yaunghon song, Jr. [10] proposed a spectral element analysis method was verified with exact analytical solution and results of the standard finite element methods. Chang Tao, Jr. [11] investigated of free and forced vibration of fiber metal laminated cracked beams subjected to moving load. Continuous wavelet transform which used for detection of location of cracked was an accurate method for damage detection. In this investigation a new model of a cracked composite beam under dynamic moving load with differential equation involves complicated terms is solved. Then the dynamic behaviors of beams under the moving loads are presented. The influence of load speed and force frequency on the forced deflection of the cracked composite beams is studied.

## 2. Theoretical Model

An Euler-Bernoulli beam with an open crack located at position x1 and a dynamic moving load with constant speed is shown in the Fig 1. The beam divided into two segments with length of L1 and L2.

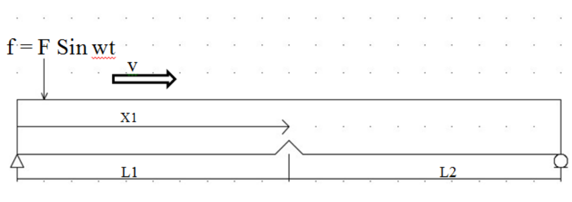


**Figure 1.** Euler-Bernoulli Beam with an Open Crack

The governing equation for the whole beam with uniform cross section, is

$$D\frac{\partial^4 w}{\partial x^4} + I_o\frac{\partial^2 w}{\partial t^2} = f(t)\delta(x - vt)H(\frac{L}{v} - t) \quad (1)$$

Where w=w(x,t) is the deflection of the beam, δ(x) and H(x) are the Dirac and Heaviside function hich can be defined as

$$\int_{-\infty}^{\infty} \delta(x)dx = 1, \quad \begin{cases}\delta(x) = 0, & for\ x \neq 0 \\ indefinite & for\ x = 0\end{cases} \quad (2)$$

$$H(x) = \begin{cases}0, & x < 0 \\ 1, & x > 0\end{cases} \quad (3)$$

$I_o$ is the inertia of the beam as

$$I_o = b\int_{-\frac{h}{2}}^{\frac{h}{2}} \rho^{(j)}dz \quad (4)$$

Where $\rho^{(j)}$ is the material density for the jth layer, h and b is the height and width of the beam, respectively. D is the reducing bending stiffness of the beam which is defined as

$$D = D_{11} - \frac{B_{11}^2}{A_{11}} \quad (5)$$

and

$$\{A_{11}, B_{11}, D_{11}\} = b\int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{Q}_{11}^{(j)}\{1, z, z^2\}dz \quad (6)$$

$\bar{Q}_{11}^{(j)}$ is the stiffness coefficient of the jth layer.

The rotational spring with sectional flexibility model for the crack and the Euler-Bernoulli theory is used for each segment. The governing equation for the free vibration can be written as

$$D\frac{\partial^4 w_i}{\partial {x_i}^4} + I_o\frac{\partial^2 w_i}{\partial t^2} = 0, \quad i = 1,2 \quad (7)$$

Method of the separation of variables is used to determine the natural frequencies.

$$w_i(x,t) = \emptyset(x)e^{i\Omega t} \quad (8)$$

By substituting Eq. (8) in Eq. (7)

$$\emptyset_i''''(x) - \beta^4\emptyset_i(x) = 0 \quad (9)$$

Where

$$\beta^4 = \frac{I_o\Omega^2}{D} \quad (10)$$

Equation (9) can be solved for crack location at $x_1$ to find the function of ∅ for each segement

$$\emptyset_1 = A_1\sin(\beta x) + B_1\cos(\beta x) + C_1 Sinh(\beta x) + D_1 Cosh(\beta x) \quad (11)$$

$$\emptyset_2 = A_2\sin(\beta(x - x_1)) + B_2\cos(\beta x(x - x_1)) + C_2 Sinh(\beta x(x - x_1)) + D_2 Cosh(\beta x(x - x_1)) \quad (12)$$

The boundary conditions for a simply supported beam are

$$w_{(0,t)} = w''_{(0,t)} = w_{(L,t)} = w''_{(L,t)} = 0 \quad (13)$$

The rotational spring model is used at $x=x_1$

$$w_1(x1,t) = w_2(x1,t),\ w''_1(x1,t) = w''_2(x1,t),\ w'''_1(x1,t) = w'''_2(x1,t),\ w'_2(x1,t) - w'_1(x1,t) = \vartheta L w''_2(x1,t) \tag{14}$$

Where $\vartheta$ sectional flexibility for a single-sided crack beam can be written as

$$\vartheta = 6\pi \bar{d}^2 f(\bar{d})\left(\frac{h}{L}\right) \tag{15}$$

$$f(\bar{d}) = 0.6348 - 1.035\bar{d} + 3.7201\bar{d}^2 - 5.177\bar{d}^3 + 7.553\bar{d}^4 - 7.332\bar{d}^5 + 2.4909\bar{d}^6 \tag{16}$$

$$\bar{d} = \frac{d}{h} \tag{17}$$

Where d is the crack depth.

The $\beta$ and coefficients of $A_1$, $B_1$, $C_1$, $D_1$, $A_2$, $B_2$, $C_2$, $D_2$ can be find by Substituting equation (13) - (17) in equations (11) and (12) as

$$\begin{cases} B_1 = 0 \\ D_1 = 0 \\ A_1 \sin(\beta L_1) + C_1 \sinh(\beta L_1) - B_2 - D_2 = 0 \\ -A_1 \sin(\beta L_1) + C_1 \sinh(\beta L_1) + B_2 - D_2 = 0 \\ -A_1 \cos(\beta L_1) + C_1 \cosh(\beta L_1) + A_2 - C_2 = 0 \\ -A_1 \cos(\beta L_1) + C_1 \cosh(\beta L_1) - A_2 - \beta\vartheta L B_2 - C_2 + \beta\vartheta L D_2 = 0 \\ A_2 \sin(\beta L_2) + B_2 \cos(\beta L_2) + C_2 \sinh(\beta L_2) + D_2 \cosh(\beta L_2) = 0 \\ -A_2 \sin(\beta L_2) - B_2 \cos(\beta L_2) + C_2 \sinh(\beta L_2) + D_2 \cosh(\beta L_2) = 0 \end{cases} \tag{18}$$

Eq. (18) can be written as

$$[\mathrm{S}(\beta)]\{\mathrm{A}\} = 0 \tag{19}$$

Where $\{\mathrm{A}\} = \{\mathrm{A}_1, \mathrm{B}_1, \mathrm{C}_1, \mathrm{D}_1, \mathrm{A}_2, \mathrm{B}_2, \mathrm{C}_2, \mathrm{D}_2\}^{\mathrm{T}}$ and $[\mathrm{S}(\beta)]$ is 8×8 matrix. For non-trivial solutions of Eq. (19)

$$|S(\beta) = 0| \tag{20}$$

The nonlinear Eq. (20) is solved to determine Eigen values $\beta j$ and modes $\phi j$. The coefficient vector {A} are chosen such that

$$\int_0^L m_p \emptyset_i \emptyset_j dx = \delta_{ij} \quad \begin{cases} \delta_{ij} = 1 \ for\ i = j \\ \delta_{ij} = 0 \ for\ i \neq j \end{cases} \tag{21}$$

The modal representation is used for a simply supported beam subjected to the moving dynamic force with constant speed

$$w(x,t) = \sum_{j=1}^{\infty} \Phi_j(x) q_{j(t)} \tag{22}$$

Where $\Phi j(x)$ is the Eigen function of jth mode

$$\Phi_j(x) = \emptyset_{1j}(x)[H(x) - H(x - x1)] + \emptyset_{2j}(x - x1)[H(x - x1) - H(x - L)] \tag{23}$$

and qj(t) is the jth modal amplitude.

$$\ddot{q}_j(t) + \omega_j^2 q_j(t) = f(t)\Phi_j(vt) \tag{24}$$

Where f(t) is dynamic force

$$f(t) = F\sin(\omega t) \tag{25}$$

Where F and $\omega$ are force amplitude and frequency, respectively. Newmark method is used to solve Eq. (25).

## 3. Experimental Setup

The experimental method that used in this paper is shown in the Fig 2. An automatic system setup is considered for consistency of the results. A dynamic force from an exciter is applied to the specimen. The exciter held by mounting fixture; move over the surface of the specimen with a controlled constant speed from the start to end point. The typical load cell and accelerometer are used to measure the force and acceleration, respectively. The dimensions of test specimen are selected as the same dimensions used in the numerical solution which are (900mm×100mm) with thickness of 5.5 mm. The material of the specimen is laminated composites with matrix of epoxy and carbon fiber. The composites made of twenty layers with fiber angle of zero. A Module is used to transfer the data from the accelerometer and load cell in order to analyse.

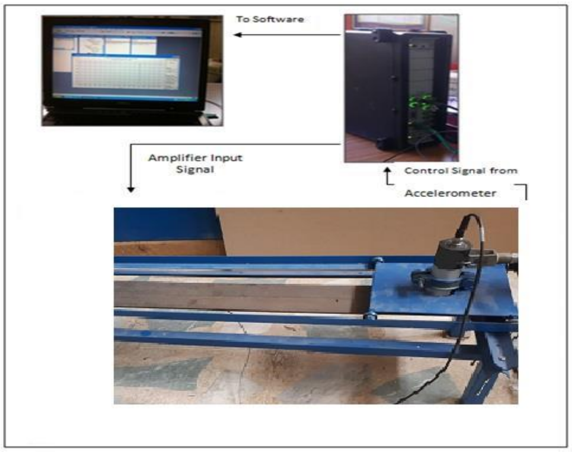


**Figure 2.** Vibration Measurement Scheme

## 4. Result and Discussion

Some numerical results are compared with the experimental data in order to validation of the present method. A cracked beam with length of L=900mm, cross sectional height h=5.5 mm, and width of b=100mm is sedelcted. Crack is located at x1=400mm with crack depth ratio of d/h =0.2. The beam that used in this article is a laminated composite Beam with twenty layer of epoxy matrix and carbon fiber and has the inertia $I_0$=0.8756 and reducing bending stiffness of D= 231.44.

The four first modes of the beam are used in modal expansion to obtain response of the system. Fig 3 shows the normalized mode for four lowest mode of simply supported cracked Beam.

The lowest four natural frequencies are: f1=31.46 Hz, f2= 126.13 Hz, f3= 283.36 Hz, and f4= 504.16 Hz. From the experiment the four first natural frequencies are measured as: f1=30 Hz, f2= 129 Hz, f3= 287 Hz, and f4= 511 Hz.

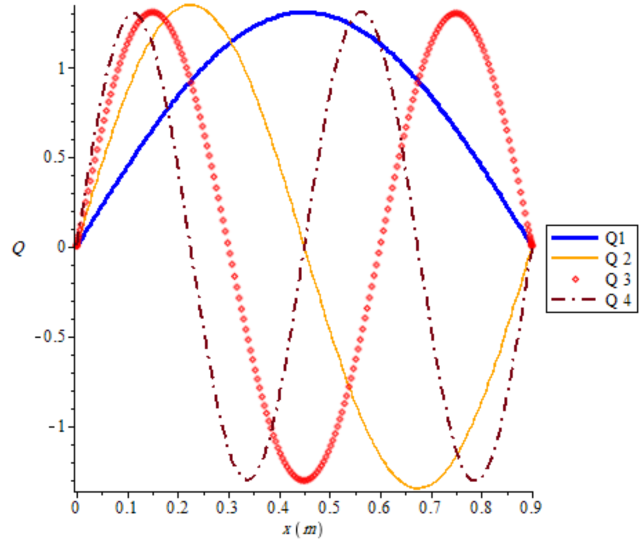


**Figure 3.** Four Lowest Modes of a Simply Supported Cracked Beam

Fig 4 illustrate the response deflection Beam at different frequency of moving dynamic load which obtained from numerical solutions and experiments at point with distance of 450 mm far from the right support, when the crack located at position of 400 mm with a crack depth ratio of 0.2. The force in sinusoidal wave with amplitude was 20 N.

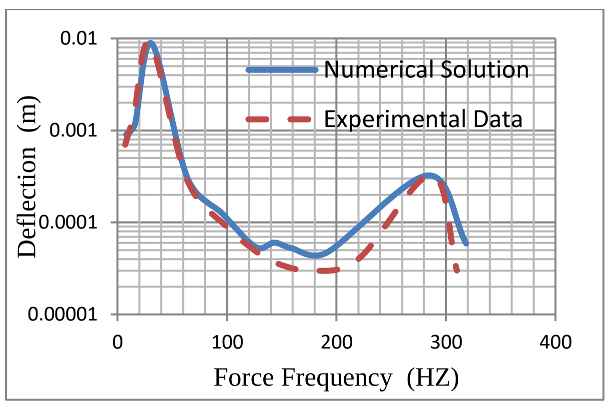


**Figure 4.** Numerical Solution and Experimental Data for Deflection at Different Force Frequency (x1=400mm, d/h=0.2, F=20N)

It is resulted that the numerical solution is satisfactory compared to the experiment.

For cases of cracked composite beams with different velocity of the moving load, Figs 5 and 6 show the forced deflection for two different frequency ω=31.85 HZ and ω= 15.92 HZ, respectively.

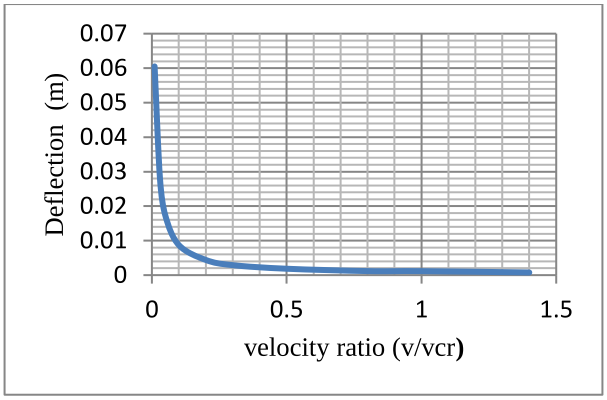


**Figure 5.** Forced Deflection at Different Velocity Ratio (ω=31.85 Hz , x1=400mm, d/h=0.2, F=20N)

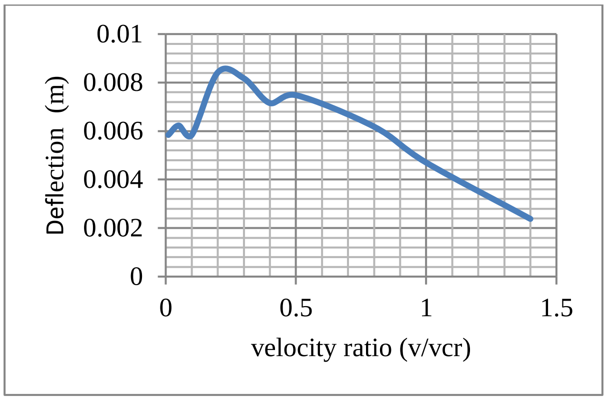


**Figure 6.** Forced Deflection at Different Velocity Ratio (ω=15.92 Hz , x1=400mm, d/h=0.2, F=20N)

The critical speed $v_{cr}$ of a simply supported beam is obtained by equating the time period of the first mode to the time required to pass through a double length of the beam.[10]

$$v_{cr} = 2f_1L \quad (26)$$

The forced deflections are decreased with increasing the speed ratio, but with different behavior with repect to the force frequency value. When the frequency is equal to 15.92 Hz the curve shows a maximum value near speed ratio of 0.2, but there is not maximum value for frequency of 31.85 Hz. It can be explained that the time is not enough for beam to reach the maximum deflection at relatively high speed of the moving load and this more effective when the force frequency is nearly to resonance frequency.

Fig 7 shows the effect of the crack depth on the forced response of simply supported composite beams.

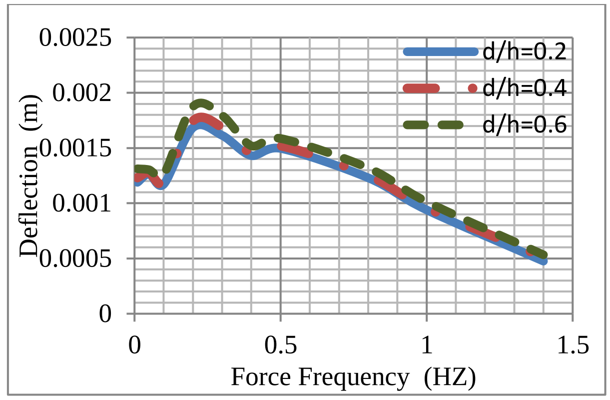


**Figure 7.** Forced Deflection at Different Velocity Ratio and Crack Depth Ratio (ω=15.92 Hz , x1=400mm, F=20N)

There is observed that the forced deflection of the cracked composite beams increased with increasing the the crack depth ratio for different velocity ratio under dynamic moving load.

Fig 8 represented the deflection for different crack position of the composite beams.

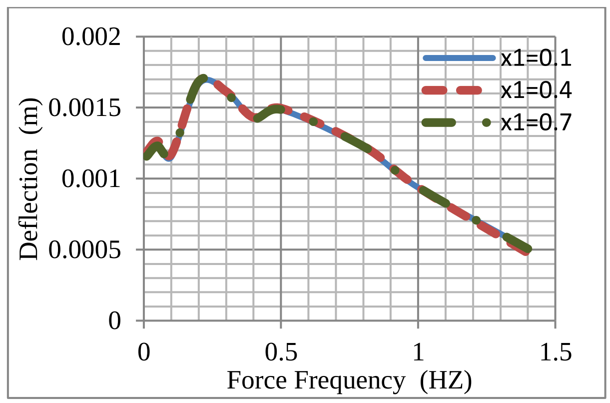


**Figure 8.** Forced Deflection at Different Velocity Ratio and Crack Position (ω=15.92 Hz , d/h=0.2, F=20N)

It is obvious from the Fig 8, the position of the crack has no important effect on the dynamic deflection of a simply supported composite beams.

## 5. Conclusion

The study presents the dynamic behavior of the cracked composite beams subjected to a dynamic moving load. By investigating a composite simply supported beam with a single crack, the following conculusions are obtained.

1- The numerical solution showed good agreement compared to experimental results.

2- The forced deflections are decreased with increasing the speed ratio, for the resonance case.

3- The graph of relation between the forced deflection and speed ratio showed a maximum value at speed ratio of 0.2 when the force frequency was below the first natural frequency.

4- The forced deflection increased with increasing the crack depth ration at different speed of moving dynamic load.

5- There is not important effect of crack position on the forced deflection of a cracked beam.

## Nomenclature

| | |
|---|---|
| $D$ | reducing bending stiffness |
| $I_o$ | inertia of the beam |
| $b$ | beam section width |
| $h$ | beam section height |
| $w$ | beam deflection |
| $\delta(x)$ | Dirac function |
| $H(x)$ | Heaviside function |
| $Q_{11}$ | stiffness coefficient |
| $\vartheta$ | sectional flexibility for a single-sided crack beam |
| $d$ | crack depth |
| F | force amplitude |
| $\omega$ | force frequency |
| $v_{cr}$ | critical force speed |
| $L$ | beam length |

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