location and crack length on vibration behavior can be identified. Furthermore, crack face displacements and crack opening displacements are calculated to study the fracture problem induced by the resonant frequencies. The vibration characteristics of an edge-cracked composite plate have been studied in [3] by using experimental analysis. The AFESPI method was adopted to investigate the vibration characteristics of a composite plate containing an edge crack. The results show that edge crack has significant effect on the modal shapes and stress intensity factors, due to the change in the induced by resonant vibration behaviour.

Free vibration analysis of damaged composite laminated plate at completely free boundary condition was studied in [4] by using finite element analysis (FEA) and experimental modal analysis (EMA). Carbon/epoxy composite AS4/PEEK was used to fabricate rectangular symmetrical laminates with [(0) 16] and [(0/90) 4]. The effect of surface crack at the location near the center of the laminated plate on the vibration characteristics was studied, where the natural frequencies and mode shapes obtained from FEA were verified by EMA. A good agreement between these two results was obtained.

The vibration behavior of a piezoelectric composite plate with cracks was analyzed in [5] by using principle of minimum energy. Dynamical model was established to study the effect of cracks and piezoelectric materials on the mode shapes of rectangular aluminum plates with and without cracks. It was shown that the strain mode is very sensitive to the presence of cracks rather than the displacement mode. This approach can be expected to detect damages of piezoelectric composite plates. The crack identification technique in plate structures was investigated in [6] by using kurtosis analysis. The vibration modes were analysed for a simply supported thin isotropic rectangular plate containing a crack parallel to one of its edges with arbitrary lengths, depths and locations. Both location and length of the crack can be accurately determined by the abrupt change in the spatial variation of the derived kurtosis analysis.

The free and forced vibration analysis of laminated composite plates and shells using strain shell element were performed in [7]. The effect of damping on the forced vibration analysis and natural frequencies of laminated composite plates and shells subjected to arbitrary loading were investigated by using FEA. Also, an iterative hybrid technique of boundary element method (BEM) and distributed dislocation method (DDM) were introduced in [8] for solving two-dimensional crack problems. The technique decomposes the problem into (n + 1) subsidiary problems where n is the number of crack branches. The stress distribution induced in the cracked plate can be defined for various boundary conditions.

Large amplitude vibration of laminated composite shells with cutout was investigated in [9], by using FEA of eight-noded isoparametric quadrilateral element. Cylindrical shell shows mostly hard spring behavior whereas spherical shell shows both hard spring and soft spring behavior with the increase of amplitude ratios for different cutout sizes, radii of curvature and thickness parameters. At a particular value of the amplitude ratio, the frequency ratio of shells is governed by interactive effects of stiffness and mass. Linear and non-linear free vibration analysis of laminated composite plates were studied in [10] based on third-order shear deformation theory with large amplitude dynamic analysis. Moreover, the vibration and buckling problems were solved in [11] for a laminated plate with a crack on edge and centrally located internal crack. The buckling mode shapes are influenced by the location and size of the crack.

The free vibration analysis of laminated composite rectangular plate was discussed in [12] using finite element method with nine- nodded isoparametric plate-bending element. Two types of an effective mass lumping scheme with rotary inertia of isotropic and fibre reinforced laminated composite plates were studied. Numerical examples of isotropic and composite rectangular plates having different fiber orientations angles, thickness ratio and aspect ratio have been solved. The vibration frequiencies of antisymmetric angle-ply laminated thin square composite plates having different boundary conditions were evaluated in [13], by using numerical analysis. The fiber orientation, number of layers, and boundary conditions have significant effects upon the natural frequencies of antisymmetric laminated composite plates.

From the previous literature, it is very important to study the effect of central crack on the vibration characteristics of laminated composite plates. Free vibration analysis can provide valuable information for damage detection and evaluation of resonant frequencies. In the present paper, free vibration analysis of a clamped laminated composite plates from all edges with central cracks has been studied based on FEA by using a well-known computer program ANSYS. The effect of various plate parameters such as number of layers, thickness, angle of fiber orientation and crack length have been considered. The cracks are as narrow, elliptical and rhombic shapes at the center of the plate, and are modeled by sawing cuts in the specimen with different length. The results obtained in the form of natural frequencies for various parameters. Some of results are compared with those available in the published literature to show the accuracy of the present analysis.

2. Theory

A square laminate composite plate with crack at the center has a uniform structure and constant thickness, while the cross section is varied at the lines through the cracks. Which modeled by sawing cuts in the specimen. Differential equation of harmonic bending vibration for laminated thin composite plate with natural frequency (ω) having side length (a and b), thickness (h), average mass density (ρ) and crack length (c) can be written in Cartesian co-ordinates (x, y) in terms of flexural displacement (u) as follows [14]:

$$D(\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4}) + \rho h \frac{\partial^2 u}{\partial t^2} = 0 \qquad (1)$$

Substituting

$$u(x, y, t) = U(x, y) e^{j\omega t} \qquad (2)$$

Gives

$$D(\frac{\partial^4 U}{\partial x^4} + 2\frac{\partial^4 U}{\partial x^2 \partial y^2} + \frac{\partial^4 U}{\partial y^4}) - \rho h \omega^2 U = 0 \qquad (3)$$

For clamped plate from all edges, the boundary conditions are:

$$\begin{aligned} &u(0, y, t) = 0 && u(x, 0, t) = 0 \\ &u(a, y, t) = 0 && u(x, b, t) = 0 \\ &\frac{\partial u}{\partial x}(0, y, t) = 0 \quad , && \frac{\partial u}{\partial y}(x, 0, t) = 0 \\ &\frac{\partial u}{\partial x}(a, y, t) = 0 && \frac{\partial u}{\partial y}(x, b, t) = 0 \end{aligned} \qquad (4)$$

Assumed mode shapes to be:

$$U(x, y) = \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad (5)$$

This satisfies the partial differential equation and the boundary conditions. So, the natural frequency of orthotropic simply supported plate is:

$$\omega_{mn} = \pi^2 [(\frac{m}{a})^2 + (\frac{n}{b})^2] \sqrt{\frac{D}{\rho h}} \qquad (6)$$

where, m, n = 1,2,3, …

The constitutive relationship for a homogenous orthotropic lamina in a state of plane stress, as shown in Fig 1(a), is [15]:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} \qquad (7)$$

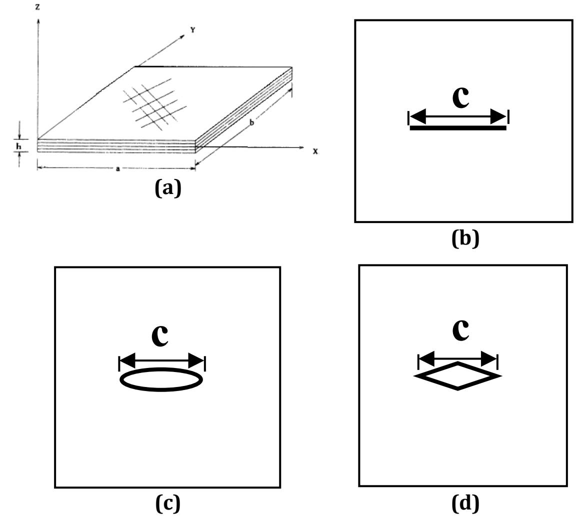


Figure 1. Geometric models: (a)Laminated plate without crack. (b)Plate with central narrow crack. (c) Plate with central elliptical crack. (d) Plate with central rhombic crack.

Where:

$$Q_{11} = \frac{E_{xx}}{1 - \upsilon_{xy}\upsilon_{yx}} \quad \cdots \quad Q_{21} = \frac{\upsilon_{yx} E_{yy}}{1 - \upsilon_{xy}\upsilon_{yx}}$$
$$Q_{22} = \frac{E_{yy}}{1 - \upsilon_{xy}\upsilon_{yx}} \quad \cdots \quad Q_{33} = G_{xy} \qquad (8)$$
$$Q_{12} = \frac{\upsilon_{yx} E_{xx}}{1 - \upsilon_{xy}\upsilon_{yx}}$$

Because of the requirement for $Q_{12} = Q_{21}$, it can obtain:

$$\upsilon_{yx} E_{xx} = \upsilon_{xy} E_{yy} \qquad (9)$$

In terms of volume fractions where $V_f + V_m = 1$, it can be obtain:

$$E_{xx} = E_f V_f + E_m V_m$$
$$E_{yy} = E_m \frac{[E_f + E_m + (E_f - E_m)V_f]}{E_f + E_m - (E_f - E_m)V_f}$$
$$\upsilon_{xy} = \upsilon_f V_f + \upsilon_m V_m \qquad (10)$$
$$G_{xy} = G_m \frac{[G_f + G_m + (G_f - G_m)V_f]}{G_f + G_m - (G_f - G_m)V_f}$$

3. Finite Element Modeling:

The ANSYS 5.4 finite element program [16] has been used to study the free vibration analysis of laminated composite plates with central cracks of different shapes, as shown in Fig. 1(b, c and d). For this purpose, the key points were first created and then the segments were formed. The lines were combined to create an area. Fine meshes of element type shell 99 with element size 0.005m with the corresponding layers, laminate thickness and angle of fiber orientation, where one quarter model was simulated due to symmetric geometry. Modal analysis with subspace method has been used to extract the modes for boundary conditions as a clamped plate from all edges.

4. Results and Discussions:

In the present formulation, the finite element method has been used for the analysis of laminate plate vibration using a computer package ANSYS. In order to demonstrate the accuracy and applicability of the present simulation, laminated composite square plates without any cracks have been analyzed and compared with the published results for both simply supported and clamped edges. A clamped thin (a/h=100) and thick (a/h=10) laminate composite plates with three layers (0/90/0) laminates have been compared with Won-Hong and Sung-Cheon [6] for the first five natural frequencies as explained in Table (1).

The second comparison is with S. Latheswary et al. [10] for the variation of non-dimensional fundamental frequency versus the width to thickness ratio with two kinds of boundary conditions as shown in Fig. (2).

Table 1. Non-dimensional natural frequencies of square laminate plates with clamped boundaries.

Mode No.	Ref. [6] a/h		Present Work a/h	
	10	100	10	100
1	17.7006	32.9806	16.90	32.99
2	24.7230	41.0807	22.86	41.11
3	33.2763	59.2342	32.74	59.54
4	36.2089	85.9108	35.56	86.21
5	37.7803	87.5651	37.32	87.65

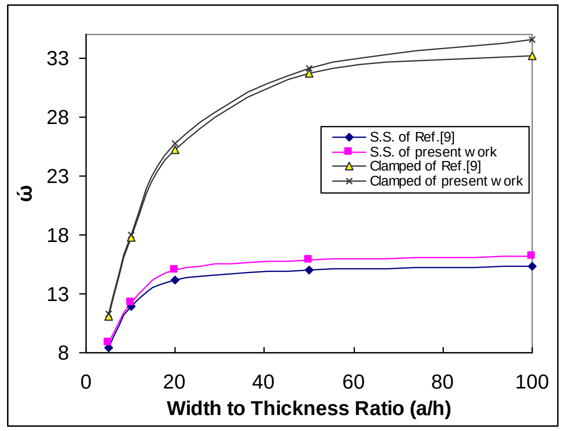


Figure 2. Variation of fundamental frequency with the width to thickness ratio as compared with S. Latheswary et al. [10]

The analyze used four-layer (0/90/0/90) laminates with simply supported and clamped edges. The results have been compared for frequency parameter calculated as non-dimensional form with good agreement results. The first six mode shapes of the laminate composite thin plate (0/90/0/90) with a central crack and without are shown in Figs. (3) to (6), given by present study as a deformation in z-direction. Then the simulation includes different crack length (c) with three shapes, angle of fiber orientation and number of layers for both thin (a/b=100) and thick (a/h=10) laminate composite plates. The effect of these various parameters on the fundamental frequency of vibration is studied by considering square laminates of side (a=b=25cm) and having the following material properties: $E_1=25E_2$, $G_{12}=G_{13}=0.5\ E_2$, $G_{23}=0.2E_2$, $\upsilon_{12}=\upsilon_{23}=\upsilon_{13}=0.25$ and $\rho= E_2$. The results are presented in a non-dimensional form, where:

$$\varpi = \frac{\omega a^2}{h}\sqrt{\frac{\rho}{E_2}} \qquad (11)$$

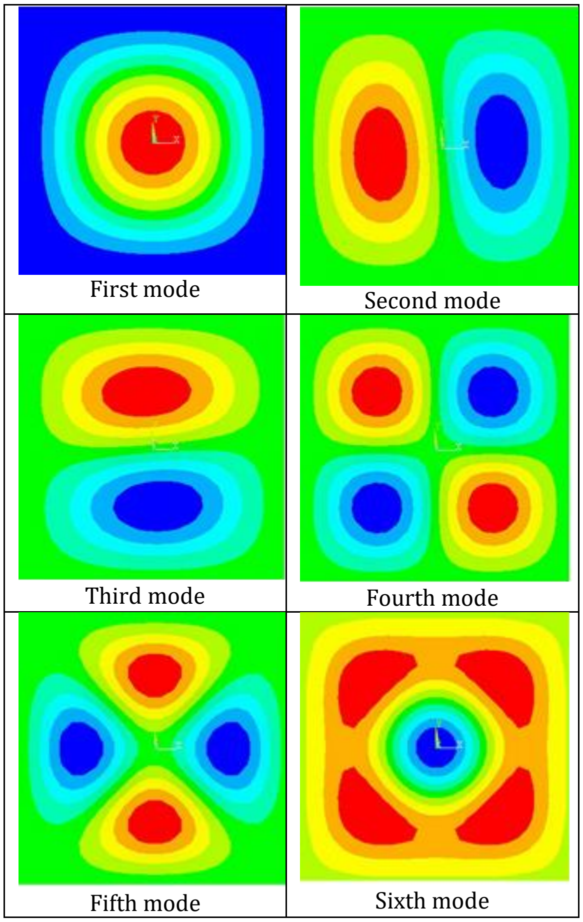
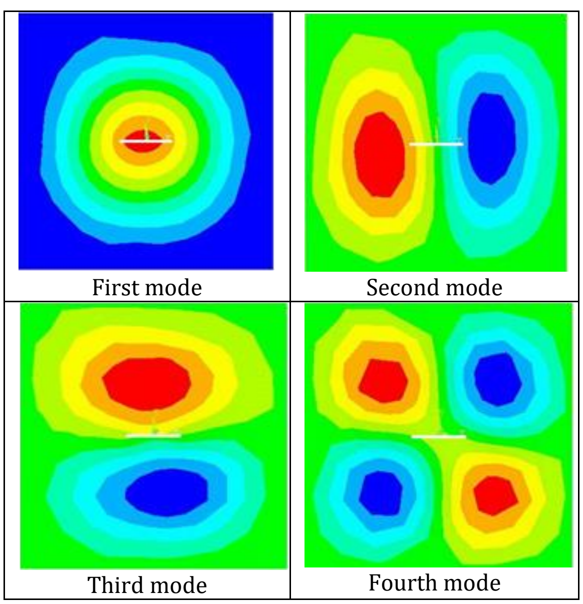


Figure 3. The first six modes of laminate clamped composite plates without crack



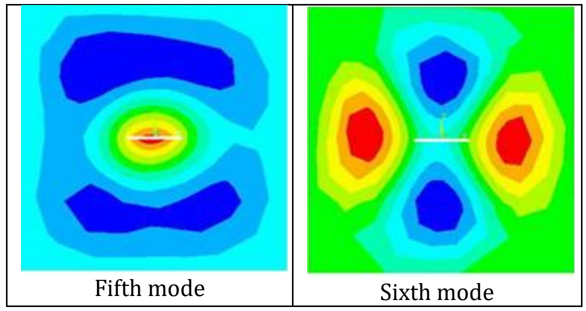


Figure 4. The first six modes of laminate clamped composite plates with narrow central crack

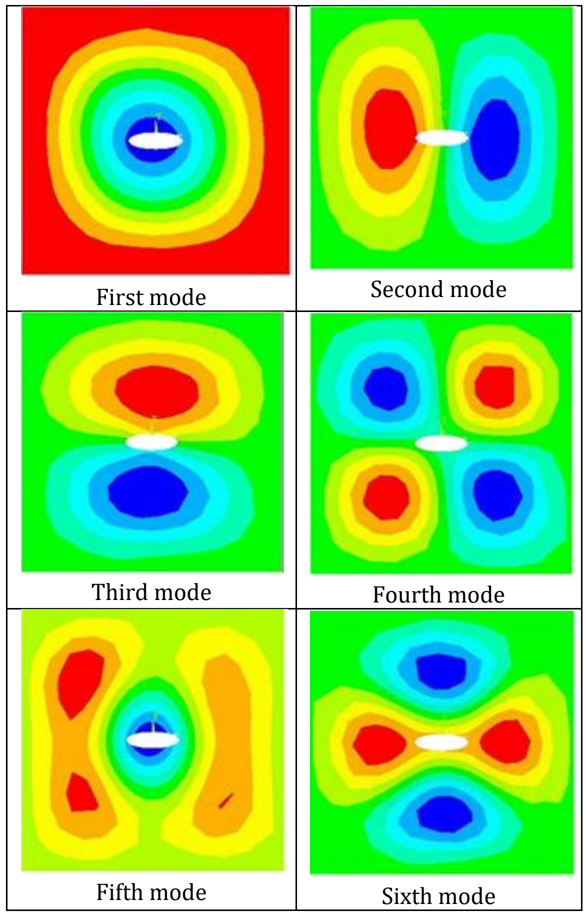
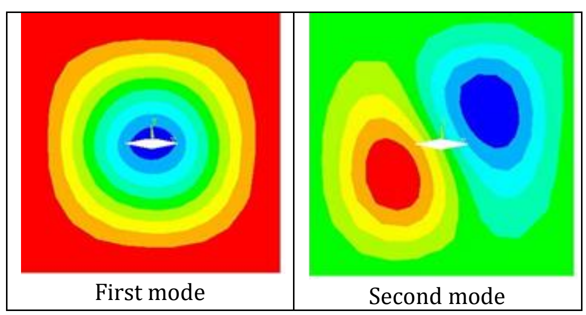


Figure 5. The first six modes of laminate clamped composite plates with elliptical central crack



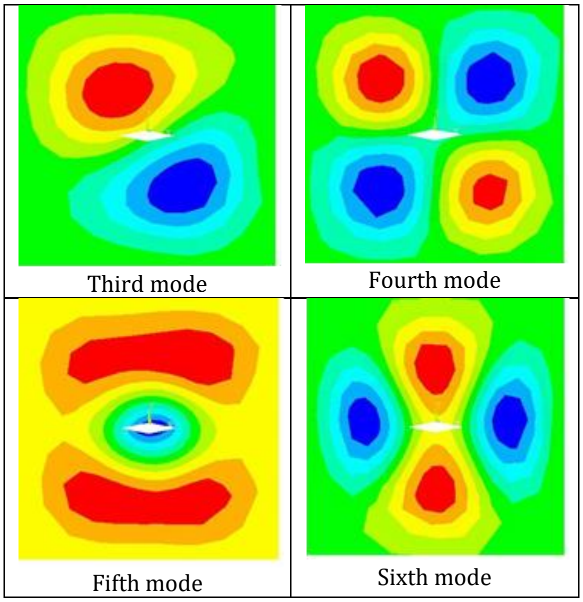


Figure 6. The first six modes of laminate clamped composite plates with rhombic central crack

4.1. Effect of Crack Length

A clamped edge of anti-symmetric four layers (0/90/0/90) with three central crack shapes for thin (a/h=100) and thick (a/h=10) plates have been analyzed. Figs. (7) and (8) show the dependence of non-dimensional frequency (ώ) with varying of crack length to width ratio (c/a) for three different cracked plates.

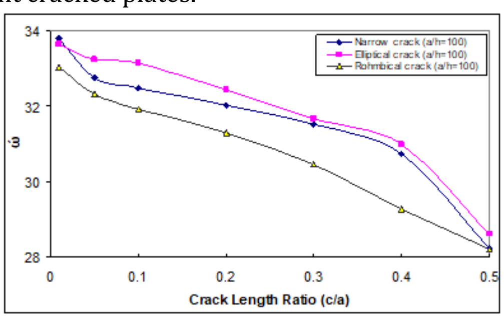


Figure 7. The variation of fundamental frequencies with crack length for thin plate (a/h=100)

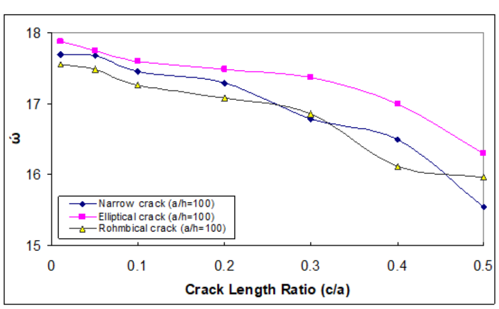


Figure 8. The variation of fundamental frequencies with crack length for thick plate (a/h=10)

The frequencies vary inversely as the crack length increases. This phenomenon can be expected because the rigidity of the cracked plate decreases as the crack length increases.

4.2. Effect of Number of Layers:

The cross-ply and angle-ply laminates (0/90/…) for clamped edges having width to thickness ratio (10 and 100) with different shapes of central crack (c/a=0.2) and without crack are analyzed to study the effect of number of layers on the fundamental frequency. The vibration results are shown in Figs. (9) and (10), where the number of layers increase the fundamental frequency increase and after 4-layers there is a small change in the non-dimensional natural frequency. The presence of crack gives low natural frequency that changed with the crack shape because of less stiffness of the laminated plate

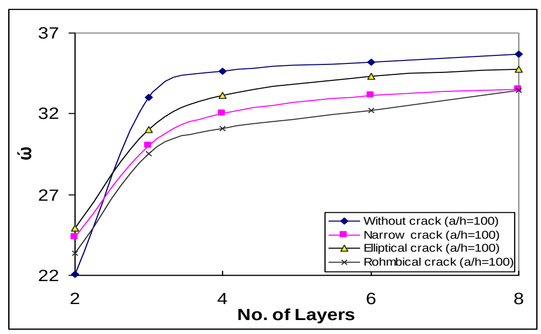


Figure 9. The variation of fundamental frequencies with number of layers for thin plate (a/h=100)

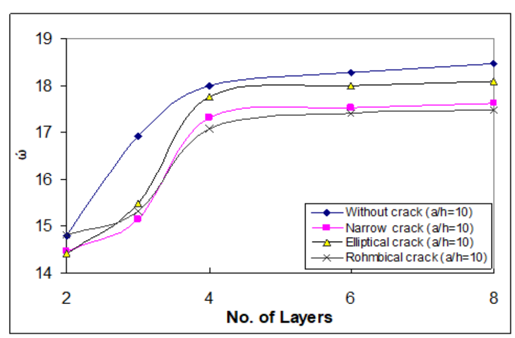


Figure 10. The variation of fundamental frequencies with number of layers for thick plate (a/h=10)

4.3. Effect of Fiber Orientation:

Four-layers anti-symmetric (α/-α/α/-α) laminates with angle of fiber orientation varying from (0° to 45°) for (a/h=10 and 100) with different shapes of crack (c/a=0.2) and without crack are analyzed. The change in the fiber orientation angle from (0° to 45°) leads to an increase in the fundamental frequency of vibration in both cases of thick (a/h=10) and thin (a/h=100) plates as shown in Figs. (11) and (12). The difference in the fundamental frequencies of vibration for cracked and uncracked plates being less for higher values of (α).

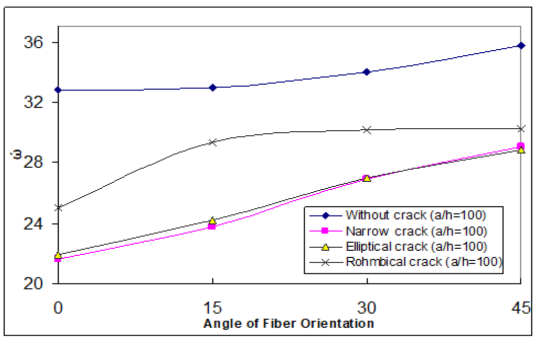


Figure 11. The variation of fundamental frequencies with fiber orientation for thin plate (a/h=100)

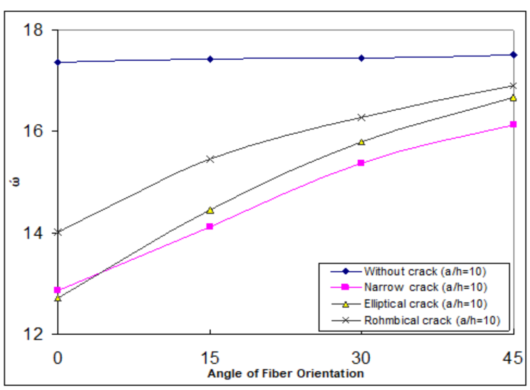


Figure 12. The variation of fundamental frequencies with fiber orientation for thick plate (a/h=10)

5. Conclusions

It is important to predict the natural frequencies of laminate composite plate with central crack because cutouts at the centre are commonly used as access ports for mechanical and electrical systems. The undesirable vibrations may cause sudden failures due to effect of resonance due to uneven shape of cutouts. The present results could lead to the following conclusions:

- The obtained results refer only to orthotropic, square plates with clamped at all edges.
- Studying the dynamic behavior of plate with a central crack can provide valuable information for the fields of vibration analysis and fracture mechanics.
- The edge boundary conditions of the plate play significant role in its resonant frequencies and the mode behaviour.
- The natural frequencies are depended on the size and shape of central crack that decreased with increasing crack length due to reduce its rigidity.
- The effect of number of layers is found to be insignificant beyond four layers of laminated plate.
- The change in angle of fiber orientation from 0 to 45 leads to an increase in the fundamental frequency of vibration in both thick (a/h=10) and thin (a/h=100).
- The variation in the fundamental frequency increases as a/h increases, but this increase is small beyond a/h=20 (as a thick plate).
- It is found that the present results are very close to the published available.

Nomenclature

u	flexural displacement.
h	thickness of plate.
E	Young modulus.
U(x,y)	mode shapes.
m, n	mode numbers.
M	bending moment.
V	volume fraction.
D	bending rigidity.
ω	natural frequency.
ώ	non-dimensional natural frequency.
ρ	density
υ	Passion ratio
α	angle of fiber orientation.

Acknowledgements

The author would like to acknowledge the University of Anbar, Iraq.

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