# Proportional Odds Nonparametric Accelerated Life Test for Reliability Prediction: An overview

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#### ABSTRACT

One way of obtaining information about reliability of units is to accelerate their life by testing at higher levels of stress (such as increasing elevated temperatures or voltages). Predicting the lifetime of a unit at normal operating conditions based on data collected at accelerated conditions is a common objective of these tests. Different models of accelerated life testing are used for such extrapolations. Two statistical based models are widely used: parametric models which require a prior specified lifetime distribution, and nonparametric models that relax of the assumption of the life testing based on the odds function and show that it gives a more accurate reliability estimates than proportional hazard model. This paper will concentrate on the models of proportional odds nonparametric accelerated life test for reliability prediction.

Keywords: Accelerated life test, accelerated failure time, proportional hazard model.

# **1.INTRODUCTION**

The failure time data are generally used for product reliability estimation. Failures of highly reliable units are rare and other information should be used than traditional censored failure time data. One way of obtaining this complementary reliability information is to apply the methods of the so-called Accelerated Life-Testing (ALT). The ALT has been recognized as a necessary activity to ensure the reliability of electronic products used in military, aerospace, automotive and mobile (cellular, laptop computers) applications, from which one can see that accelerated testing of electronic products offers great potential for improvements in reliability life testing, to obtain required reliability results for the products more quickly than under normal operating conditions, given in terms of covariates (stresses), which could be time-varying.[1] Two major non parametric models are widely used; the proportional odds model based on the odds function show that more accurate than the proportional hazard model.

The common objective of ALT is to predict the life time of a unit at normal operation condition [1]. The reliability of unit can be obtained by through acceleration the life by testing at a high level of stress such as increasing temperatures and voltages. The rapid advances of technology and high quality improvement efforts, being established hence many products are so reliable that traditional life tests are not feasible in estimating the lifetimes of these products. In such circumstances, accelerated tests are widely used to shorten the life of products or hasten the degradation of their performance. The aim of such tests is to collect data quickly so that desirable information on product life or on performance under usual use can be obtained in a reasonable time by appropriate modeling and analysis [2].

Models of accelerated life testing are used for such extrapolations. Two statistics based models are widely used: parametric models which require a prior specified lifetime distribution, and nonparametric models that relax of the assumption of the life time

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distribution. In parametric models, the failure times distribution, such as exponential or Weibull, etc, must be identified in advance [3].

# 2. PREDICTION IN RELIABILITY

The concept of deriving mathematical models which could be used to predicate reliability, are in the same way as models are developed and used in other scientific and engineering fields [4]. The accuracy of the used inference procedure (models) has a profound effect on the reliability estimate and the subsequent decisions regarding system configuration, warranty, and preventive maintenance schedules[5]. From this point of view it is necessary to develop models which reflect the reliability of units to be accurate. Elsayed in 1996 [5] classifies inference procedures into three types:

- Statistics-based models
- Physics-statistics-based models
- Physics-experimental-based models.

Furthermore, he classifies the statistics-based models into two categories: parametric models and nonparametric models. Many researchers (Such as Huang and Jin) are concentrated on statistical based models, so that this paper gives attention on this model only.

In parametric models, it is usually assumed that there is only a scale transformation between the failure times at different stress levels (accelerated failure time) with the distribution type remaining unchanged. In other words, the shape parameters of the failure-time distributions are equal for all stress levels but the scale parameters may vary. The analysis of failure-time data with applied stresses under accelerated conditions often involves complex and not wellknown failure-time distributions. A good ALT model therefore needs to be flexible in order to deal with such natural complexity and robust to unknown failure-time distributions [6]. To avoid making additional assumptions that would be difficult to test and validate, nonparametric models appear to be attractive alternatives. Nonparametric models relax the assumption of the failure-time distributions and thus are essentially distribution-free [3].

# 3. PARAMETRIC ACCELERATED LIFE TESTING (ALT) MODEL

An accelerated test environment is created typically by increasing the level of one or more of the stress variables (temperature, voltage, etc.) to values which are higher than those at normal operating conditions. It is assumed that the environment is defined by a single stress variable, and the extension to the multivariable case is straightforward [6].

To introduce some notation, let  $z_i$  denote the level of the stress variable where (i) stress level ( 1, 2, 3 ... k) at the  $z_3 > z_2 > z_1$  accelerated test environment and assume that test will be conducted at k accelerated levels of the stress variable which are specified in advance. As noted before, the main objective is to make inferences about failure behavior of items at the use stress environment, z0, based on the data from the k accelerated testing environments where  $z_1 > z_0$ .

It is assume that under the i<sup>th</sup> accelerated test data, the failure behavior of the items can be described as specific parametric model to compute parameters of their distribution and statistical goodness of fit tests should be applied to test the fit to the assumed distributions [4]. The time-consuming evaluation of a product's lifetime often prevents manufacturers from meeting market requirements within the time allotted for product development. Accelerated life tests have been widely employed for decades to reduce the time needed for assessing a

product's lifetime or reliability. They involve exposing the product to an environment that is harsher than the normal operating one in order to expedite the failures and thereby reduce the required test time[6].

Typical accelerating variables used to stress test units are temperature, voltages, mechanical load, thermal cycling, humidity, and vibration. The data are then analyzed assuming a statistical lifetime distribution and a pre-specified stress-life relationship to estimate the product lifetimes at the use-condition [6]. Huang.T and Jiang.T in 2008 [7] used Weibull and log-logistic models to validate two distributions of accelerated failure time model when failure time distribution follow mix of Weibull and log-logistic distribution. One major and essential type of parametric models is accelerated failure time (AFT) which is reviewed below.

## 4. PARAMETRIC ACCELERATED FAILURE TIME (AFT) MODEL.

AFT model assumes that covariates (or applied stresses) act multiplicatively on the failure time, or linearly on the log (failure time). Denote the failure time of a unit under a vector of covariates z by T, and the failure time under normal stresses (covariates) by  $T_0[7]$ .

The AFT model assumes that: 
$$T_0 = \exp(z'\beta T)$$
 (1)

The hazard function of the AFT model can be expressed in terms of a baseline hazard function  $\lambda_0$  as:

$$\lambda(t; z) = \lambda_0 \exp(z'\beta)t \exp(z'\beta)$$
(2)

The AFT models are equivalent to the class of linear models for:

$$Y = \ln T = \ln T_0 + z'\beta \tag{3}$$

with its error density function defined corresponding to what  $\lambda_0(t_0)$  implies. Ordinary linear regression methods can be used to estimate  $\beta$ , but it is difficult to include censored data in these methods[3]. The estimation methods are parametric if the function  $\lambda_0(t_0)$  is specified with the appropriate error density function. On the other hand, nonparametric methods could be implemented without specifying the form of  $\lambda_0(t_0)$  [3]. However, the AFT model has not been widely used in practice, mainly due to the difficulties in computing the semi parametric estimators of the mentioned methods, even in situations when the number of covariates is relatively small [8]. For high-dimensional covariates it is even more difficult to apply these methods, or their regularized versions, especially when variable selection is needed along with estimation[8].

## 5.NONPARAMETRIC ACCELERATED LIFE TESTING (ALT) MODEL

Methods have been developed for measuring and comparing ALT data when no assumption is made as to the form of the underlying distribution. These called nonparametric or distribution free statistical methods [9]. The nonparametric regression model for response Yi is:

$$Y_i = \mu + f(z_1, z_2, \dots, z_{(N)}) + \mathcal{C}_i$$
(4)

i = 1;...;k, stress level , where  $\mu$  is the intercept, f is the unknown function of covariates zi and  $\varepsilon i$  is error. With many predictors, choosing an appropriate subset of the covariates is a

crucial, and difficult step in fitting a nonparametric regression model. Several methods exist for curve fitting and variable selection for multiple nonparametric regressions [10]. Nonparametric regression techniques are useful in obtaining a smooth fit to noisy data, to describe the relationship between response variables and statistical -independent variables [9]. The failure time data at accelerated conditions might be the result of mixtures of different distributions and it becomes difficult to use standard models for reliability prediction. Under this condition, a generalized model is likely to provide more accurate estimates if properly developed [7].

Aranda-Ordaz (1981) [11] proposes a parameter family which makes the proportional hazard model and the proportional odds model special cases for medical purposes when some conditions are met. Parameter family of Aranda-Ordaz does not used for ALT in industrial applications until this time. There are two adequate types of nonparametric models based on ALT are reviewed.

# 5-1.Cox's Proportional Hazard (PH) model

Cox (1972) [12] considered the introduction of predicator /explanatory covariates into such models which means the hazard may be thought of as proportional to the instantaneous probability of an event at particular time. This model is used by multiply the hazard function by a function of the covariates. Fisher &Lin in 1990 [13] shows that the covariates in this model may also be used in models in which the underlying survival (reliability) curve has fully parametric form such as Weibull distribution. One main issue in time to event data analysis is to study the dependence of the survival time T on covariates  $z = (z(1), \ldots, z(n))$ . This task is often simplified by using the Cox's proportional hazard model, where the log hazard function is the sum of a totally unspecified log baseline hazard function and a parameterized form of the covariates. [14].

Leemis in 1995 [15] gives the Proportional Hazards model which important in the analysis of life data. The model has been widely used in the biomedical field and recently there has been an increasing interest in its application in reliability engineering. In its original form the model is nonparametric, i.e., no assumptions are made about the nature or shape of the underlying failure distribution.

The Proportional Hazards (PH) model of Cox (1972) is a widely accepted nonparametric model. It has been successfully used in survival analyses and in modeling ALT. However, due to its strong assumption of hazard rate proportionality it does not allow hazard functions under different stress levels to cross. Regardless of the distribution, there are two usual assumptions:

- The shape parameter is independent of the applied stresses, and
- The scale parameter is a function of the stress variables.

For example, when temperature is the only accelerating stress in the test, the functional form for the scale parameter is derived from the Arrhenius equation [3].

$$\lambda(\text{temp}) = \gamma_0 \exp \{-E_a / (k_B(\text{temp } \circ C + 273.15))\}$$
(5)

where  $\lambda$  is hazard function, temp = temperature (°C) + 273.15 is the temperature in Kelvin and  $k_B = 1/11605$  is Boltzmann's constant in units of electron volts per K. The reaction activation energy,  $E_a$ , and  $\gamma_0$  are characteristics of the product or material being tested.

The Arrhenius model and other simple acceleration models result in what Meeker and Escobar (1999) [16] call a Scale Accelerated Failure Time (SAFT) model, in which the time to failure T(z) is related to the time to failure time  $T(z_0)$  at environment conditions  $z_0$  through the relationship. [16]

$$T(z) = T(z_0)/AF(z)$$
: (6)

Where AF(z) accelerated failure at z stress. Cox and Oakes in 1984 [17] developed the Proportional Hazards model assumes that the failure rate of a unit is equal to the product of a baseline failure rate,  $\lambda_0(t)$ , which is a function of time only, and a positive function (also known as link function) g(z, c), independent of time, which incorporates the effects of a number of covariates (z) such as humidity, temperature, pressure, voltage, etc. The failure rate of a unit is then given by,

$$\lambda_{\lambda} \lambda(t, z) = \lambda_{0}(t) (t)(t) g(z, c)$$
(7)

With the assumption that the form of g(z, c) is known and  $\lambda_0(t)$  is unspecified.

Different forms of a link function g(z, c) can be used such as linear(1+cz), log (log(1+cz), exp (exp cz) and inverse exp (1/exp(cz)) as suggested in 200 by Mettas Mettas [18]. However, the exponential form is mostly used due to its simplicity and the hazard rate can be given by the following equation:

$$\lambda(t, z) = \lambda_0(t) \exp \sum^k c_i z_i$$
(8)

 $\begin{array}{l} \lambda \left( t,\,z \right) \text{ is the Failure rate (hazard rate)} \\ \lambda_0(t) \quad \text{is the baseline failure rate} \\ z_i \quad \text{is the row vector consisting of the covariates} \\ c_i \quad \text{is the column vector consisting of the unknown parameters} \\ \text{For } i = 1, \ 2, \ 3, ..., k \quad \text{is the stress types and levels} \end{array}$ 

The Cox Proportional Hazards (PH) model is the most common approach for statistical modeling of survivor ship data, and has been widely applied in epidemiological studies as given by Cox and Oakes in 1972 [17].

$$\lambda(t, z) = \lambda_0(t) \exp(c z^{\mathrm{T}})$$
(9)

where z is a particular dimensional vector of covariates, c is regression parameters and  $\lambda$  (t, z) is conditional hazard function of random variable and  $\lambda_0(t)$  as an arbitrary baseline hazard function.

Although  $\lambda_0(t)$  is not specified, the only unknown parameter in the partial likelihood function is C, which can be estimated by maximizing the partial likelihood function or the log-partiallikelihood function ln(L(C)) numerically [14]. The PH model is based upon the hazard function and summarizes treatment effects in terms of the ratio of age-specific mortality rates in two treatments (i.e., the hazard ratio). Hazard ratios have been an important tool in medical research, but it is clear that biology of aging researchers prefer to visualize experimental results in terms of survival curves, rather than hazard functions. This has likely prevented the PH model from being widely used in experimental aging research, and has compelled many investigators to summarize treatment effects in terms of percent change in median or mean lifespan. But Kelin in 1997 [24] emphases that PH model is widely used for accelerated life testing because of its relative easiness of estimating procedures and reliability practitioners to interface to the statistical software.

One explanation is that the PH model does not generate an intuitive summary statistic that is interpreted in terms of survivorship as discussed by Hodder and Keene in 2002 [21]. The main assumption of the proportional hazards model is that the ratio of hazard rate of two units under two stresses level Z1, Z2 is constant over time this lead to that the hazard rates are proportional to the applied stress level [23]. Kalbfleisch and Prentice (2002) [19] used parametric proportional hazard models. Parametric hazard models often assume using the linear form which is simple and useful in practice but may be too rigid for complicated problem. More flexible models allow  $\eta(z)$  to vary in some infinite dimensional space.

The PH model has only seldom been used to evaluate treatment effects on mouse survivorship Conti et al., 2006 [20]. The generalized PH models are such as additive model and extend linear hazard regression model are developed. Wang in his doctorial dissertation research in 2004 [25] developed a generalized Extended Linear Hazard Regression (ELHR) model with linear time-varying coefficients to estimate reliability under normal operating conditions using failure time data obtained from accelerated conditions, the model deals with different effects such as time-varying coefficients effect and time-scale changing effect done by Wang [25]. Extensive simulation experiments demonstrate that the ELHR model provides more accurate reliability estimates than those obtained by the existing ALT models. The experimental works for the model is accomplished to study the reliability of time-dependent-dielectric-breakdown of thermal oxides on n-type 6H-SiC using laboratory data.

One major drawback which limiting uses PH model is that the difficulty to analyze failures under the effect of fixed hazard rate over time.

#### 5-2.Proportional Odds(PO) model

Brass in 1971 [26],observed that the ratio of two hazard rates for two groups under different stress levels ( for example smoker and non smokers ) is varying with time and more complicated course as relative therefore the PH model is not suitable for such case .There in 1974 [27] he proposed more realistic model called proportional odds model. The odds function can be define as the odds on failure of unit at time t or the ratio between the probability of failure and the probability of reliability (survival). Huang in 1995 [28] used the odds model as logit function as the link where:

$$logit(x) = log(x/(1-x))$$
(10)

While in the Cox model, log(-log) is the link (which means that the covariates linked to lifetime). Dabrowska and Doksum in 1988 [29,30], proposed estimation procedure called Generalized Odds rate (GOR) model, their estimate are consistent and asymptotically normal while the PO is special cases for two sample case. They considered the generalized odds function for a random failure time T as follows:

$$\Lambda_{\rm T}(t/c) = \begin{cases} 1/c[1-(1-F(t))^{\circ}c], & c>0\\ -\log[1-F(t)], & c=0 \end{cases}$$
(11)

Where  $\Lambda_T(t/0)$  is cumulative hazard rate, F(t) is the cumulative distribution function for failure time T and c is binary value.

 $\Lambda_T(t/0)$  is cumulative hazard rate, where  $\Lambda_T(t/1)$  is odds of the failure before t. For c other than 1,  $\Lambda_T(t/c)$  also has interpretation as an odds function for some situation.

Pettitt in 1984 [31] used an approximation method to estimate proportional odds model for failure time data using ranks for failure time instead of true failure observations. This approximation is based on a Taylor series expansion of the marginal likelihood. This estimation procedure was based on ranks of failure time therefore will not provide well tested approach to estimate the reliability under design or operation stress level or conditions. Murphy in 1997 [32] used the profile likelihood estimates and

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proved that the maximum profile likelihood estimator for stress coefficients is consistent, asymptotically normal and efficient.

The calculation of maximum profile likelihood estimator depends on numerical algorithms which Murphy used the modification of Newton – Raphson algorithms. From the model of Bennett in 1983 [33,34] and Pettitt in 1984 [35] in survival analysis context. Can easily be seen that in the two sample setup (i.e., z = 0 or 1), the ratio of the hazard rates for the two groups converges to 1 over time. Consequently, Bennett [33] has suggested using proportional odds structure to model effective cure. Rossini and Tsiatis in 1996 [36] gives some other applications of this model and can be used in the analysis of environmental health data. Zhang in 2007 [37] developed the proportional odds model in accelerated life testing based on the odds function and shows that it gives a more accurate reliability estimates than proportional hazard model when the underlying lifetime distribution is log-logistic [34]. Sundarm in his paper in 2009 [38] studied the estimation of proportional odds model with time dependent covariates based on semi parametric inference. The proposed estimators include a class of minimum distance estimators defined through weighted odds function. These estimators are shown to be strongly consistently estimated.

#### **6. CONCLUSION**

Traditional reliability modeling is based on parametric models assumed that the failure times can be fit with one distribution model. Nonparametric models are not assuming any type of distribution; therefore these models are more accurate than conventional one. The proportional hazard model (Cox model) is adequate approach for nonparametric model for medical and biomedical applications because of simple covariates (binary represent) in these fields. In reliability prediction proportional odds model is more useful than first one because of the ability to model any type of covariates and time dependent covariates. In reliability modeling the competing causes in the accelerated life tests, there are two distinct situations: one when the causes of failure remain the same at all the stresses, the other when the causes of failure are affected by the stress in the sense that the probability of failure due to a specific cause depends on the stress. A lot of works has to be done in the field of reliability in order to establish good models to be used in a general form rather than a specific form due to huge types of items and applications.

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عرض بحثي التخمين الموثوقية باستخدام النسبية الاحادية بدون توزيع لاختبار العمر المعجل				
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		الخلاصة.		

بيانات موثوقية المنتجات يمكن الحصول عليها باختبار العمر المعجل تحت مستويات إجهاد عالية. نماذج مختلفة تستخدم لاختبارات العمر المعجل للتقريب .هناك نموذجان إحصائية شائعة الاستخدام ذو المتغيرات وبدون متغيرات أي بدون توزيع احتمالي للعمر . نموذج النسبية الأحادية المستند على نموذج بدون متغيرات لاختبار العمر المعجل يبين انه أكثر دقه من نموذج المخاطرة النسبية. هذا البحث يركز على النموذج المعجل بدون متغيرات لتخمين الموثوقية.