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Cyclic Torsion Behavior of Prestressed Concrete Beams

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ABSTRACT

The nonlinear finite element analysis has become an important tool, for the structural design and assessment of prestressed reinforced concrete members. However, design and assessment of torsion are still done with simplified analytical or empirical design methods. This paper presents results from a numerical analysis using the ANSYS finite element program to simulate a prestressed concrete beams subjected to static and cyclic torque. The eight- node brick elements SOLID65 are used for the idealization of concrete while the reinforcements are idealized by using 3D spar element LINK8. The steel plates are idealized by using three dimensional solid elements SOLID45. The results showed that the general behavior of the finite element models represented by torque- twist angle relationships show good agreement with the experimental results from the Abdullah's beams.

1. Introduction

Prestressed Concrete has been used extensively over the past decades and it has become one of the major structural building materials and it is widely used all over the world, especially in urban areas Nawy [5]. There are several methods for modeling the concrete structures through both analytical and numerical approaches. Modern computational techniques like the finite element method have been used in nonlinear analysis of reinforced and prestressed concrete structures.

The finite element method (FEM) is a numerical method which provides a tool that can accurately simulate the behavior of concrete structures. This technique that has been widely used for linear and nonlinear analysis of reinforced and prestressed concrete structures. Attention, at early stages of using finite element method, was focused on two dimensional and axisymmetrical models, but it was soon extended to include plate, shell and three-dimensional system under general states of loading.

In this paper, the ANSYS [2] finite element program is used to simulate a prestressed concrete beams tested by Abdullah [1]. This package provides a three-dimensional element (SOLID65) with the nonlinear model of brittle materials similar to the concrete material. The element features in a smeared crack analogy for cracking in tension zones is accommodated by the nonlinear material model and a plasticity algorithm to take into account the concrete crushing in compression zones. The truss element (LINK8) is used for discrete reinforcement modeling (prestressing strands, longitudinal and transverse reinforcements). However, it was assumed that perfect bonding between concrete and steel reinforcement occurs during loading.

The main objective of this study was to develop a numerical model where nonlinear material properties of prestressed concrete beams can be included in detailed analysis for cyclic torsion investigations in the future. A three-dimensional nonlinear of pre-stressed concrete beams was developed by a general purpose finite element analysis package ANSYS.

2. Finite Element Formulation

The finite element method has been adopted by a number of areas of engineering such as heat transfer and magnetic field analysis. In this study, the original application of structural engineering is considered. In this domain, the technique broadly consists of discrediting a structure into a number of small substructures. This allows the displacement or stress in these elements to be approximated, the latter being the most common approach. These elements must then be assembled in such a way that stresses are continuous across element interfaces and the internal stresses are in equilibrium both with each other and with the applied loads. The finite element method can thus be thought of as a two stage process, the first being the construction of finite elements and the second their assembly into structural matrices.

2.1. Matrix Structural Analysis

The most common problem arising in structural analysis is to determine the deflection arising from a set of static loads. If the loads at these nodes -or degrees of freedom- are defined by a vector {F} and the displacements at the corresponding nodes are similarly defined {u}, matrix stiffness [K] is required to relate the load and displacement:

$$\{F\} = [K]\{u\} \tag{1}$$

Of most interest to dynamics is similar formulation which includes inertia and damping terms:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\} \tag{2}$$

Where: [M] is the mass matrix describing the distribution of mass about the structural degrees of freedom and $\{\dot{u}\}$ and $\{\ddot{u}\}$ are the first and second derivatives of the displacement with respect to time. Notice that the force applied to the system is now a function of time. While mass and stiffness of a structure are measured and relatively easily derived, the mechanism whereby energy is lost through damping is less easily modeled. The viscous damping model represented by matrix [C] in equation (2) is commonly, but by no means exclusively, used being proportional to velocity. Structural damping is an important area of structural dynamics which has deservedly received much attention. Of most importance is that damping dominates the amplitude of vibration around resonance Ewins [3]. The undamping equation of motion from equation (2) is:

$$\{\ddot{u}\} + [K]\{u\} = \{F\} \tag{3}$$

For free (unforced) vibrations the following relation is obeyed:

$$[M]\{\ddot{u}\} + [K]\{u\} = 0 \tag{4}$$

The solution to which can be written in the form:

$$\{u\} = \{\psi\}_j e^{i\omega_j t} \tag{5}$$

where the ω_j are the resonant frequencies. Substituting back into equation (4) leads to the well known Eigen Value problem:

$$[M]\{\psi\}_j \lambda_j = [K]\{\psi\}_j \tag{6}$$

where $\left(\lambda_j = \omega_j{}^2\right)$ and $\{\psi\}_j$ can be thought of the mode shapes corresponding to the system natural frequencies $\{\omega\}_j$. While the Eigen Values have an exact relationship with the resonant frequencies, the eigenvectors are arbitrarily scaled; it is common practice to define a uniquely scaled set of eigenvectors such that:

$$[\emptyset]^T[M][\emptyset] = [I] \tag{7}$$

These results in:

$$[\emptyset]^T[K][\emptyset] = diag(\lambda) \tag{8}$$

where $[\emptyset]$ is the matrix of mass normalized eigenvectors.

2.2. Equations of Motion for Finite Elements

By using the principle of virtual work, it can develop equations of motion for any finite element. Such equations include energy equivalent stiffnesses, masses, and nodal loads for a typical element. The derivation presented therein is general and can be applied to any element type and proposed displacement field. Assume that a three-dimensional finite element with zero damping exists in Cartesian coordinates x, y, and z. at any point within the element. If the element is subjected to time-varying body forces, such forces may be placed into a vector {b (t)}. The shape functions $[\mathbf{N}]$are used to express the time-varying generic displacements at any point within the element,$\{\mathbf{U}(\mathbf{t})\}_{\mathbf{e}}$in terms of the time-varying nodal displacements, $\{\mathbf{u}\ (\mathbf{t})\}_{\mathbf{e}}$, Weaver and Johnston [10] as:

$$\{\boldsymbol{U}(\boldsymbol{t})\}_{\boldsymbol{e}} = [\boldsymbol{N}]\{\boldsymbol{u}(\boldsymbol{t})\}_{\boldsymbol{e}} \tag{9}$$

By differentiation of the displacements, the corresponding time-varying strain$\{\boldsymbol{\varepsilon}\}_{\mathbf{e}}$, can be written as:

$$\{\boldsymbol{\varepsilon}(\boldsymbol{t})\}_{\boldsymbol{e}} = [\boldsymbol{\partial}]\{\boldsymbol{U}(\boldsymbol{t})\}_{\boldsymbol{e}} \tag{10}$$

where $[\boldsymbol{\partial}]$is the matrix of differential operators. Substituting of equation (9) into (10) yields:

$$\{\varepsilon(t)\}_e = [B]\{u(t)\}_e \tag{11}$$

where [B] is the strain-nodal displacement matrix given by:

$$[B] = [\partial].[N] \tag{12}$$

The stresses $\{\sigma(t)\}_e$ can be determined from the corresponding strains, by using the general stress-strain relationship as:

$$\{\sigma(t)\}_e = [E].\{\varepsilon(t)\}_e \tag{13}$$

where $\{\sigma(t)\}_e$ is the time-varying stress vector, given by:

$$\{\sigma(t)\}_e = [\sigma_x \quad \sigma_y \quad \sigma_z \tau_{xy} \quad \tau_{yz} \quad \tau_{zx}]^T \tag{14}$$

And [E] is the constitutive matrix and$\{\varepsilon(t)\}_e$, is the strain vector given by:

$$\{\varepsilon\}_e = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{Bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \tag{15}$$

Sub stitution of equation (11) into equation (13) gives the stress-nodal displacement relationship:

$$\{\sigma(t)\}_e = [E].[B]\{u(t)\}_e \tag{16}$$

The principle of virtual displacements of deformed body is used to establish the governing equations of dynamic equilibrium. It means that, if a general structural in dynamic equilibrium is subjected to a system of small virtual displacements with a compatible state of deformation, the virtual work of external actions($\boldsymbol{\delta W_{ext}}$)is equal to the virtual strain energy of internal stresses($\boldsymbol{\delta W_{int}}$), Weaver and Johnston [10] thus:

$$(\boldsymbol{\delta W_{ext}}) = (\boldsymbol{\delta W_{int}}) \tag{17}$$

For the external virtual work, an infinitesimal element with components of applied body forces $\mathbf{b}(\mathbf{t})$ and the internal body forces $\boldsymbol{\rho}\ddot{\mathbf{u}}\mathbf{dV}$ due to the acceleration$\ddot{\mathbf{u}}$. The symbol $\boldsymbol{\rho}$ in these expressions represents the mass density of the material, which is defined as the inertial force per unit acceleration per unit volume. Note that the inertial forces act in directions that are opposite to the positive senses of the acceleration. Thus, it adds the external virtual work of nodal and distributed body forces as follows:

$$\boldsymbol{\delta W_{ext}} = \boldsymbol{\delta\{u\}^T\{P_i(t)\}} + \int \boldsymbol{\delta\{U\}^T\{b(t)\}dV} - \int \boldsymbol{\delta\{U\}^T\rho\,\{\ddot{U}\}dV} \tag{18}$$

where $\{\mathbf{P_i}(\mathbf{t})\}$ is a time-varying nodal action. The internal virtual strain work is given by:

$$\boldsymbol{\delta W_{int}} = \int \boldsymbol{\partial\{\varepsilon\}^T\{\sigma(t)\}dV} \tag{19}$$

Substitution of equations (18) and (19) into equation (17) produces:

$$\boldsymbol{\delta\{u\}^T\{P_i(t)\}} + \int \boldsymbol{\delta\{U\}^T\{b(t)\}dV} - \int \boldsymbol{\delta\{U\}^T\rho\{\ddot{U}\}dV} = \int \boldsymbol{\partial\{\varepsilon\}^T\{\sigma(t)\}dV} \tag{20}$$

The above expression represents the equation of dynamic equilibrium for a general body. Therefore, by making use of equation (9), (11), and assume that$\{\ddot{\mathbf{U}}\} = [\mathbf{N}]\{\ddot{\mathbf{u}}\}$, and substitute into equation (20) to obtain:

$$\boldsymbol{\delta\{u\}^T \int \rho[N]^T[N]dV\{\ddot{u}\}} + \boldsymbol{\delta\{u\}^T \int [B]^T[E].[B]dV\{u\}} = \boldsymbol{\{u\}^T\{P_i(t)\}} + \boldsymbol{\delta\{u\}^T \int [N]^T\{b(t)\}dV} \tag{21}$$

Cancellation of $\boldsymbol{\delta\{u\}^T}$and rearrangement of the resulting equations of motion is given in equation (3.3).

where, $$[\boldsymbol{K}]_e = \int [\boldsymbol{B}]^T[\boldsymbol{E}][\boldsymbol{B}])\boldsymbol{dV} \tag{22}$$

And, $$[\boldsymbol{M}]_e = \int \boldsymbol{\rho}[\boldsymbol{N}]^T[\boldsymbol{N}]\,\boldsymbol{dV} \tag{23}$$

Matrix [K] in equation (22) is the element stiffness matrix, which contains stiffness coefficient that is fictitious actions at nodes due to unit values of nodal displacements. Equation (23) gives the form of the consistent- mass matrix, in which the terms are energy-equivalent actions at nodes due to unit values of nodal accelerations Weaver and Johnston [10]. Finally, the vector $\int[\mathbf{N}]^{\mathbf{T}}\{\mathbf{b}(\mathbf{t})\}\mathbf{dV}$ in equation (21) consist of equivalent nodal loads due to body forces in the vector$\{\mathbf{b}(\mathbf{t})\}$. Other equivalent nodal loads due to initial strains or stresses could be derived Weaver and Johnston [9], but analyses for such influences are considered to be statics problems.

2.3. Coordinate Transformation

If local axes for a finite element are not parallel to global axes for the whole structure, rotation of axes transformations must be used for nodal loads, displacements, accelerations, stiffness, and consistent masses. Thus, when the elements are assembled, the resulting equation of motion will pertain to the global directions at each node. The formulation of the elemental matrices described above has for convenience and generality been relative to a set of axes local to the element itself. To convert elemental stiffness matrices from local set of coordinates $(\bar{x}, \bar{y}, \bar{z})$ within which they were formulated into the global coordinates (x, y, z) of the structure of which they must form a part a transformation is required. This is given by:

$$\begin{Bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{Bmatrix} = [T]\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} \frac{\partial \bar{x}}{\partial x} & \frac{\partial \bar{x}}{\partial y} & \frac{\partial \bar{x}}{\partial z} \\ \frac{\partial \bar{y}}{\partial x} & \frac{\partial \bar{y}}{\partial y} & \frac{\partial \bar{y}}{\partial z} \\ \frac{\partial \bar{z}}{\partial x} & \frac{\partial \bar{z}}{\partial y} & \frac{\partial \bar{z}}{\partial z} \end{bmatrix}\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{24}$$

where, T is the transformation matrix. Forces in-elemental coordinates can similarly be transformed:

$$[\bar{F}]^T = [T][F] \tag{25}$$

Equating work done in the local coordinates with that done in global coordinates, it can be shown that:

$$[F]^T[u] = [\bar{F}]^T[\bar{u}] = [F]^T[T]^T[T][u] \tag{26}$$

Considering work done once more and now including the local elemental stiffness $[\bar{K}] = [\bar{F}][\bar{u}]^{-1}$ and the global elemental stiffness [K]:

$$[\bar{u}]_e^T[\bar{K}]_e[\bar{u}]_e = [u]_e^T[K]_e[u]_e = [\bar{u}]_e^T[T]^T[\bar{K}]_e[T][\bar{u}]_e \tag{27}$$

where $[K]_e=[T]^T[\overline{K}]_e[T]$, represents the element stiffness matrix in global coordinates.

3. Finite Element Analysis Using ANSYS

The philosophy behind the "analysis" phase is to find the mathematical model which describes the most resemblance behavior of the member. Although there are many methods of analysis for structures, no single one is ideal for all problems. This requires that the actual properties of the test piece to be taken into consideration. The appropriate parameters for the actual properties are; stiffness, mass distribution, and boundary conditions Rad [6]. The finite element method is the most popular simulation method to predict the physical behavior of systems and structures. The ANSYS software is one of the leading commercial finite element programs in the world and can be applied to a large number of applications in engineering. Finite element solutions are available for several engineering disciplines such as static, dynamic, heat flow, fluid flow, electromagnetic, and also coupled field problems. In this study, three types of analysis are used.

3.1. Static Analysis

A static analysis calculates the effects of steady loading conditions on a structure, while ignoring inertia and damping effects in equation (2). It determines the displacement, stresses, strains, and forces in structures or components caused by loads under static loading condition. A static analysis can be either linear or nonlinear. All types of nonlinearities are allowed- large deformations, plasticity, creep, stress stiffening, contact elements, and hyperelastic elements.

3.2. Model Analysis

A modal analysis is used to determine the vibration characteristics of a structure, while it is being designed. Hence, the goal of modal analysis is to determine the natural frequencies and mode shapes of a structure. The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions. Modal analysis calculated according to equation (4) where damping ignored and unforced. It is a linear analysis. In this study, using modal analysis to determine:

A. Mode Shape: a mode shape is a specific pattern of vibration executed by a mechanical system at a specific frequency. Different modes will be associated with different frequencies. The mode shape that caused torsion mode used in this study. Figure (1) was shown torsion mode.

B. Natural Frequencies: the natural frequency is the rate at which an object vibrates when it is not disturbed by an outside force. Each degree of freedom of an object has its own natural frequency. The equations to calculate the natural frequency depended on equations (4), (5), and (6). The undamped natural frequency is equal to the square root of the ratio of stiffness matrix to mass matrix ($\omega = \sqrt[2]{K/m}$). The natural frequency calculated from modal and harmonic analysis and then chosen the minimum frequency (critical) for torsion mode shape.

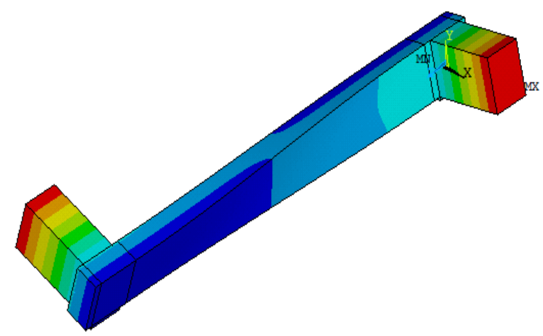

Figure 1.**Mode shape (Torsion).**

3.3. Cyclic Analysis

For calculation of cyclic analysis by ANSYS used transient analysis. Transient dynamic analysis or time-history analysis is a technique used to determine the dynamic response of a structure under the action of any general time- dependent loads. It determines the time- varying displacements, strains, stresses, and forces in a structure as its responses to any combination of static, transient, and harmonic loads.The time scale of the loading is such that the inertia or damping effects are considered to be important. The basic equation of motion solved by a transient dynamic analysis is equation (2). In this study, to determine the cyclic loading the damping effect is ignored and the force is applied as a function displacement according to equation follow Timoshenko et. al. [7]:

$$Func._{time}* = -1*(uy_{cr} + time)*Sin(freq.*time) \quad (28)$$

where, $Func._{time}$ is displacement Function of Time (mm), uy_{cr} is displacement in (Y-axis) at Cracking Torque in Static Analysis (mm), timeis time in sec.

andfreq. is natural Frequency in Modal analysis (rad/sec).* This function represents the time history for all specimens, which are given by trial and error then chooses the best time history.A transient dynamic analysis is more involved than a static analysis because it generally requires more computer resources in terms of the "engineering" time involved. There are three solution methods available to a transient dynamic analysis; full, reduced, and mode superposition.

3.3.1 Full Method

The full method does not reduce the dimension of the considered problem since original matrices are used to compute the solution. As a consequence it is simple to use, all kinds of nonlinearities may be specified, automatic time stepping is available, and all types of loads (nodal forces, imposed (non-zero) displacements, and element loads (pressures and temperatures)) may be specified. It uses full matrices, masses matrixes are not assumed to be concentrated at the nodes and finally all results are computed in a single calculation. The main disadvantage of the full method is the fact that the required solution times will increase with the size of the considered model. This method uses in this study. The procedure for analyzing nonlinear transient behavior is similar to that used for nonlinear static behavior. The main difference between the static and transient procedures is that time- integration effects. Basic information about nonlinear analysis is presented in ANSYS Structural Analysis Guide, but in a transient nonlinear analysis must specify steppedor ramped loads.If a load is stepped, then its full value is applied at the first substep and stays constant for the rest of the load step, as shown in figure (2a). If a load is ramped, then its value increases gradually at each substep, with the full value occurring at the end of the load step, as shown in figure (2b).

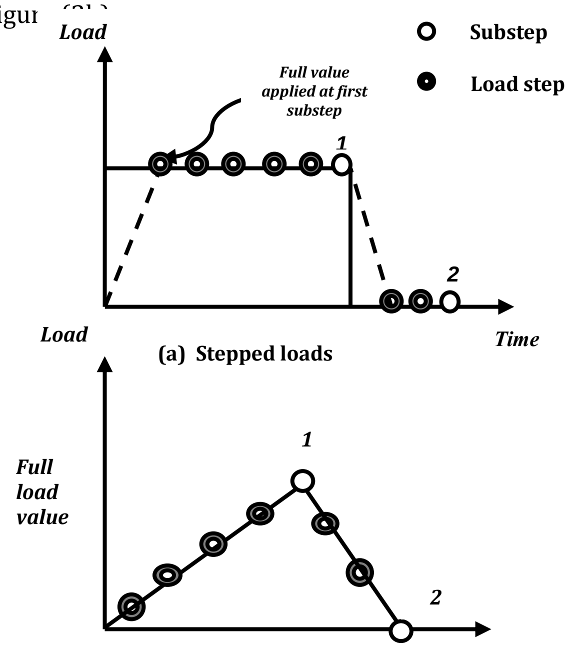

Figure 2.**stepped and ramped loads. ANSYS [2]**

3.3.2 Reduced Method

The reduced method originates from earlier years. Because of the reduced system matrices which are used to solve the transient problem, this method has an advantage when compared with the full method with respect to the required solution time. However, the user has to specify master degrees of freedom which represent the dynamic behavior as good as possible. The only nonlinearity which can be specified is node to node contact via a gap condition. However, automatic time stepping is not possible. Consequently, this method is not very popular anymore since all its disadvantages do not really compensate the advantage of lower costs in solution time.

3.3.3Modal Superposition Method

The modal superposition method usually reduces the dimension of the original problem as well since the transient analysis is finally performed in the modal subspace which has the dimension of the number of mode shapes used for the superposition. The main advantage is again the reduction of solution time. It turns out that this method is actually the most efficient one compared with the other two. The accuracy just depends on the number of mode shapes used for the modal superposition. Even if a few modes shapes are taken the requested solution time might still be less when compared with the full and the reduced method. The time step has to be chosen as constant which means that automatic time stepping is not available for this method. The solution process consists basically of two analysis, the modal analysis and transient analysis in the modal subspace. Since for most problems in structural dynamics the natural frequencies of a structure are of interest this is not really a disadvantage. Summing up, using the modal superposition technique for a transient analysis reduces not only solution time, but the user also obtains information about the natural frequencies and the undamped mode shapes, respectively.

4. Specimens Detailed

A prestressed concrete beams were cast and tested by Abdullah [1]. Four beams have been selected for present study [A1, A2, B2, and C2] all these beams subjected to cyclic torque but the beam A1 subjected to static torque; the dimensions of all specimens are geometrically similar, having rectangular cross section, of dimension (125x250x2500mm). The amount of the lateral reinforcement was varied and the loading history was similar. The beams were pretensioned eccentrically by four 10mm diameter placed inside a plastic sewage 16 mm diameter. The distribution of prestressing stresses was varied linearly from zero at the upper section to the maximum value at the lower section equivalent to 45% of ultimate compressive strength. Dimensions and Reinforcement details of the beams are given in figure (3) while table (1) shows the details of beams and material properties

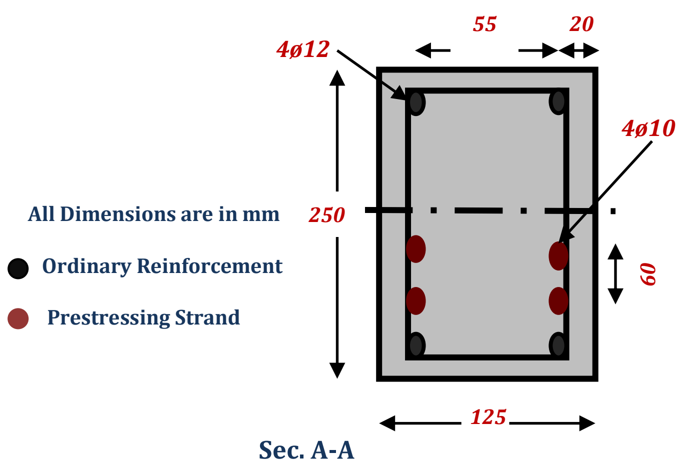

*Figure 3.**Dimensions and Reinforcement Details of the Beams (A1, A2, B2, and C2). Abdullah [1]***

Table 1. Details of beams and material properties

Beams	(f'_c) (MPa)	(f_t) (MPa)	(f_r) (MPa)	(E_c) (MPa)	(S) (mm)	Pi (kN)
A1	43.29	4.91	5.59	30720	50	76.1
A2	47.19	4.93	6.35	32250	50	82.95
B2	44.99	4.8	6.35	31550	75	79.08
C2	51.89	5.25	6.69	32710	100	91.21

5. Materials Modeling

5.1. Concrete Modeling

ANSYS software provides a three-dimensional element SOLID65 with nonlinear model of brittle materials similar to the concrete material. This element has eight nodes with three translation degrees of freedom u, v and w in the x, y and z directions respectively. Cracking of concrete and stress-strain relation in tension is modeled by a linear elastic behavior, taking to consideration tension stiffening after cracking. In the present study the ability of concrete to transfer shear forces across the crack interface is accounted for by using two different shear retention factor ($\boldsymbol{\beta}$) for cracked shear modulus, it is assumed equal to 0.15 for opened crack and 0.5 for closed crack for beams under cyclic torque while it is assumed to equal to 0.38 and 0.7 for opened and closed crack for beam A1, respectively. Many models for the stress-strain curve of concrete under uniaxial compression have been proposed in past years. The Vecchio- Collins stress- strain curve is more suitable for reinforced concrete members subjected to torsion Hsu and Mo [4]. In the present study used the Hognestad parabola relationship to determine the compressive stress $\mathbf{f_c}$ corresponding to the compressive principal strain $\boldsymbol{\varepsilon_c}$ according to Vecchio and Collins[8] and given as:

$$f_c = \begin{cases} -f_p\left[2\left(\frac{\varepsilon_c}{\varepsilon_p}\right)-\left(\frac{\varepsilon_c}{\varepsilon_p}\right)^2\right], 0 > \varepsilon_c > \varepsilon_p & (a) \\ -f_p\left[1-\left(\frac{\varepsilon_c-\varepsilon_p}{2\varepsilon_0-\varepsilon_p}\right)^2\right], \varepsilon_p > \varepsilon_c > 2\varepsilon_0 & (b) \end{cases} \quad (29)$$

where: f_p and $\boldsymbol{\varepsilon_p}$ are the peak stress and corresponding strain, respectively. The strain at peak stress is calculated as $2\mathbf{fc}^{`}/\mathbf{E_c}$, where: $\mathbf{E_c}$ is the initial slope of the parapola. The peak stress will be the cylinder strength $\mathbf{f_c'}$ which occurs at the strain $\boldsymbol{\varepsilon_0}$, for the case of uniaxial compression. The resulting multilinear isotropic stress-strain curve for concrete is shown in figure (4).

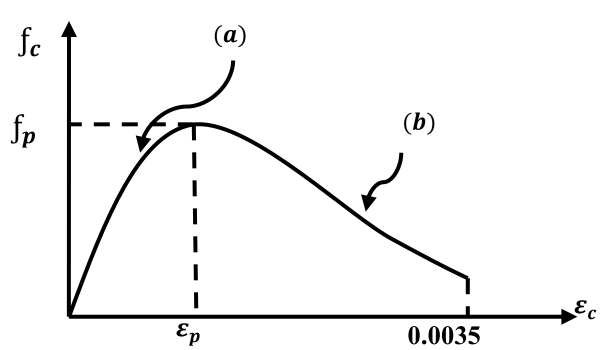

Figure 4. ***Vecchio and Collins' Stress- Strain Curve for Concrete.***

5.2. Reinforcement Modeling

Modeling of reinforcing steel in finite elements is simpler than modeling of concrete. LINK8 element was used to model steel reinforcement. This element is a 3D spar element and it has two nodes with three degrees of freedom- translations in the nodal x, y, and z direction. This element is also capable of plastic deformation.

5.3. Strands

In the present study a multilinear isotropic stress- strain is adopted for prestressing strands by using equations (31) or (32) below:

$$\varepsilon_{ps} \le 0.008 \Longrightarrow f_{ps} = 193100\,.\,\varepsilon_{ps}\ (MPa)\ \ (30)$$

$$\varepsilon_{ps} > 0.008 \Longrightarrow f_{ps} = 1930 - \frac{0.517}{\varepsilon_{ps}-0.0065} < 0.98\, f_{pu}\ (Mpa)(32)$$

The elastic modulus of prestressing strands is taken equal to 193100 MPa. The adopted stress-strain curve is shown in figure (5). The prestressing forces are input as strain in the LINK8 element that is corresponding to the allowable effective stresses of the strands.

5.4 Longitudinal and Stirrups

Only horizontal lines should be used within a tableTypical stress- strain curves for longitudinal and stirrups reinforcing bars used in concrete construction are obtained from Abdullah [1] tests as shown in figure (5), while yielding stress ($\mathbf{f_y}$) equal to 433MPa and 392 MPa for longitudinal and stirrups, respectively.

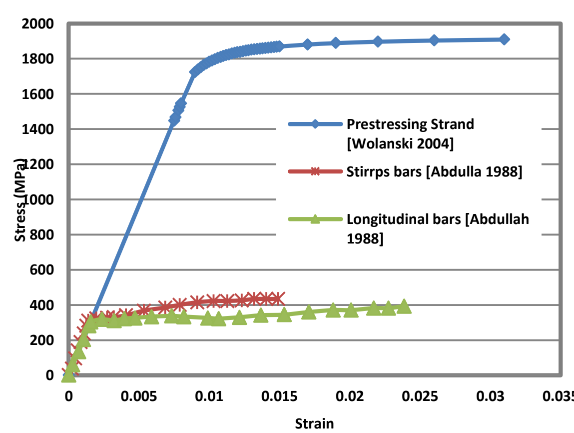

Figure 5. **Stress- Strain relationship for reinforcement.**

5.5. Steel Plates

The SOLID45 brick element used to represent the loading and anchorage steel plates, which are used to avoid local failure of concrete at the load and anchorage locations. This element is defined with eight nodes having three degrees of freedom at each node-translation in the nodal x, y, and z directions. The adopted behavior for the loading and anchorage plates are assumed as Bilinear Kinematics Hardening (BKIN). The adopted yielding stress is equal to 400MPa and hardening modulus is taken to 200MPa; while the elastic modulus is equal to 200000 MPa and Poisson's ratio is equal to 0.3.

6. Finite Element Idealization, Loading and Boundary Condition

The total effective length of the beam was considered in the finite element analysis with 2500 elements for beams (A1, A2, and C2) and with 4320 elements for beam B2. The concrete and reinforcement elements are arranged as shown in figure (6); also boundary condition of beam is shown. The beam was restrained against movement in lateral, vertical and axial directions at mid span. At free end on the lever arm, the cyclic torque is represented by an equivalentset of displacement function for time according to equation (30).

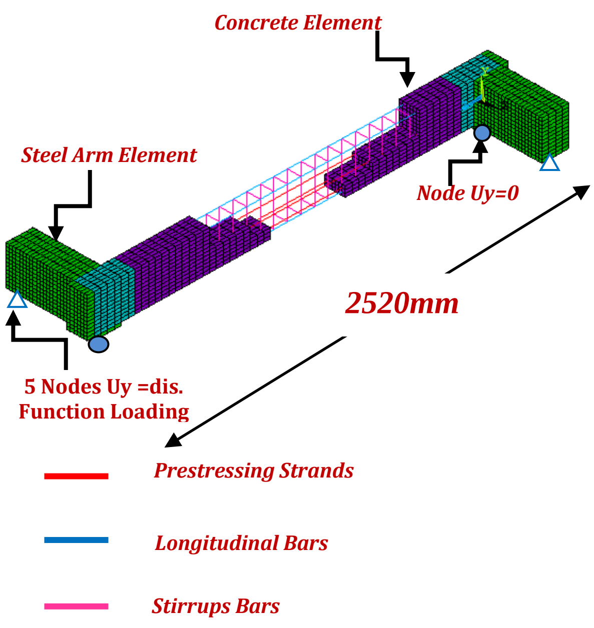

*Figure (6): **Finite Element Model for Beams***

7. Numerical Analysis and Comparison of Results

The experimental and numerical torque-twist curves obtained for these beams are shown in figures (7), (10), (13) and (16). It reveals that the finite element solution give good results compared with experimental results throughout the entire range of behavior. The numerical, the experimental ultimate torque, cracking torque, the ratio of the numerical ultimate torque to experimental ultimate torque, ratio of the numerical to experimental cracking torque and ultimate twist angle were given in table (2). Figures (8), (11), (14) and (17) show the deformation shape resulting from torsion moment by finite element analysis, while figure (9), (12), (15) and (18) show the location of cracks and their types for Beams (A1, A2, B2 and C2) respectively.The predicted ultimate torque, cracking torque and ultimate twist angle compared with the experimental Abdullah's beams are given in Table (2). It is clearly seen from figures (5), (8), (11) and (14) that the predicted results of the proposed model used in finite element analysis using ANSYS software show a good agreement with the experimental observation for torque-twist relationships and ultimate torque.

Beams	Tcr(EXP.) kN.m	Tcr(Num.) kN.m	Tcr(Num.)/Tcr(Exp.)	Tu(Exp.) kN.m
A1	8	8.2	1.025	11.42
A2	8.6	8.97	1.043	11
B2	7	6.92	0.989	10.75
C2	8.25	8.12	0.984	10.51

Beams	Tu(Num.) kN.m	Tu(Num.)/Tu(Exp.)	θ_u(Num.) / θ_u(Exp.)
A1	12	1.051	0.88
A2	11.97	1.088	0.853
B2	10.6	0.986	0.782
C2	11.71	11.42	0.62

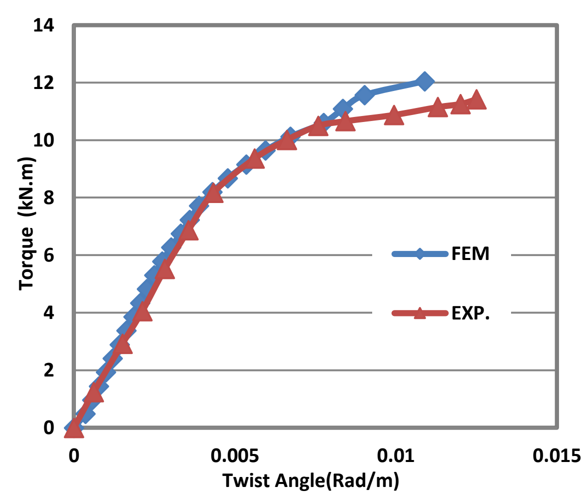

Figure 7. **Static Torque-Twist Angle Relationship for Experimental and Numerical Results for Beam (A1).**

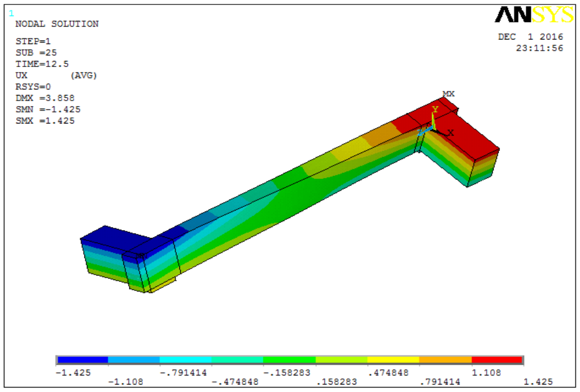

Figure 8. **Deformed Shape at Static Torque Analysis for Model (A1).**

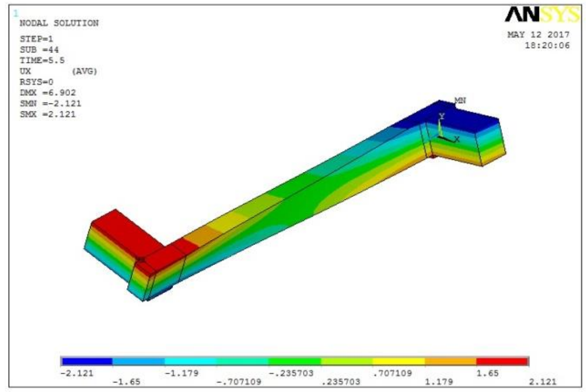

Figure 11. **Deformed Shape at Cyclic Torque Analysis for Model (A2).**

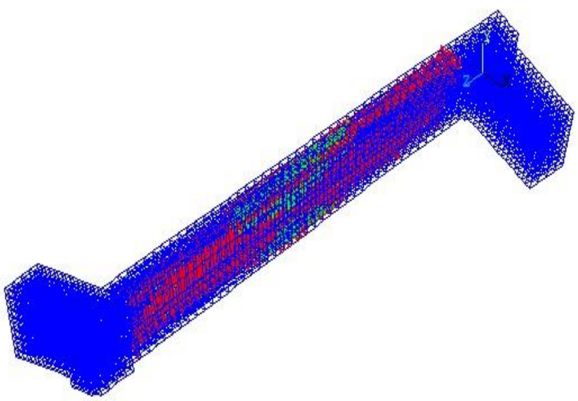

Figure 9. **Crack Pattern at Static Torque Analysis for Model (A1).**

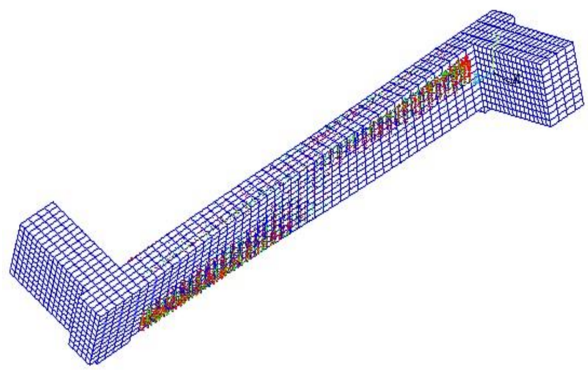

Figure 12. **Crack Pattern at Cyclic Torque Analysis for Model (A2).**

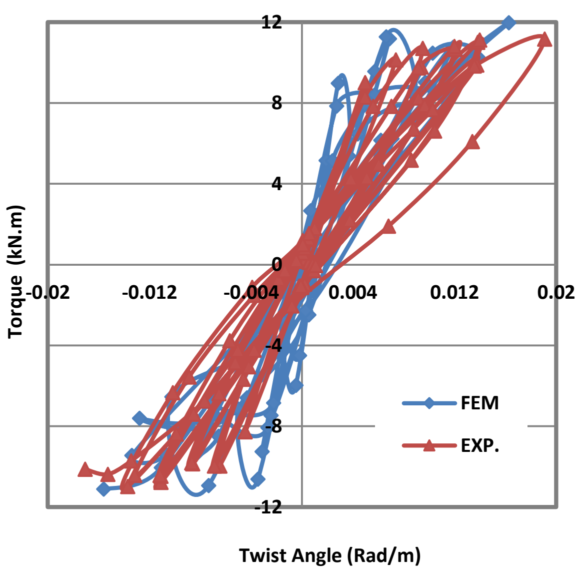

Figure 10. **Cyclic Torque-Twist Angle for Experimental and Numerical for Beam (A2).**

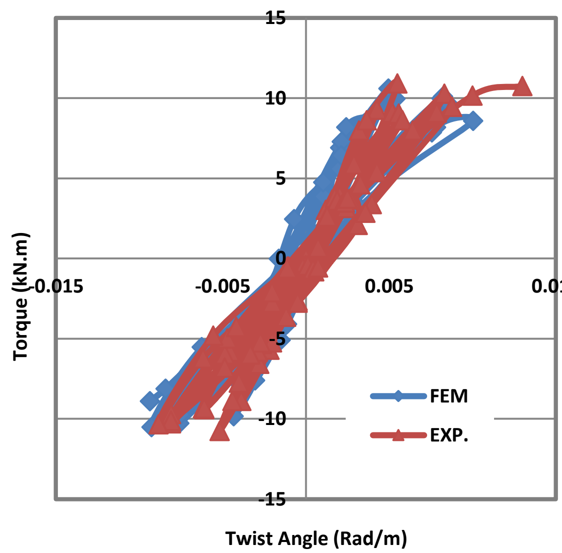

Figure 13. **Cyclic Torque-Twist Angle for Experimental and Numerical for Beam (B2).**

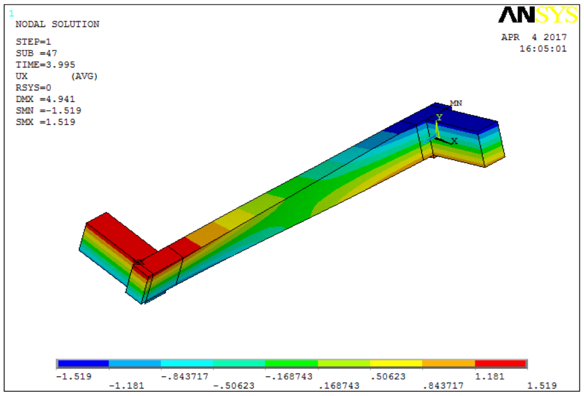

Figure 14. **Deformed Shape at Cyclic Torque Analysis for Model (B2).**

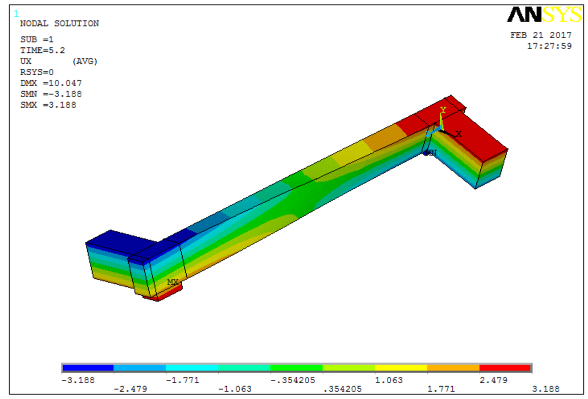

Figure 17. **Deformed Shape at Cyclic Torque Analysis for Model (C2).**

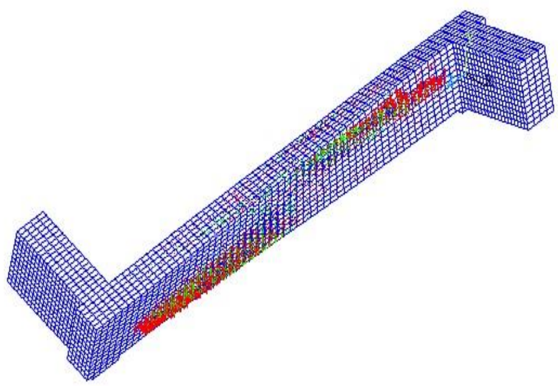

Figure 15. **Crack Pattern at Cyclic Torque Analysis for Model (B2).**

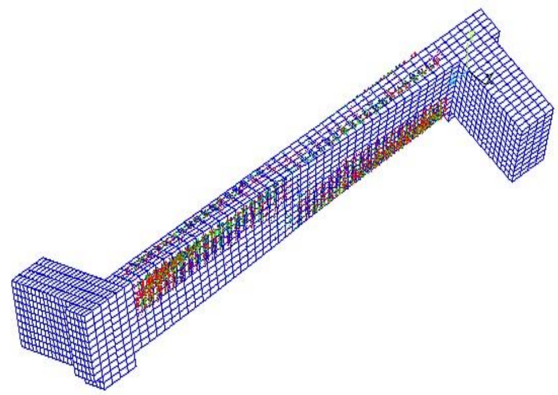

Figure 18. **Crack Pattern at Cyclic Torque Analysis for Model (C2).**

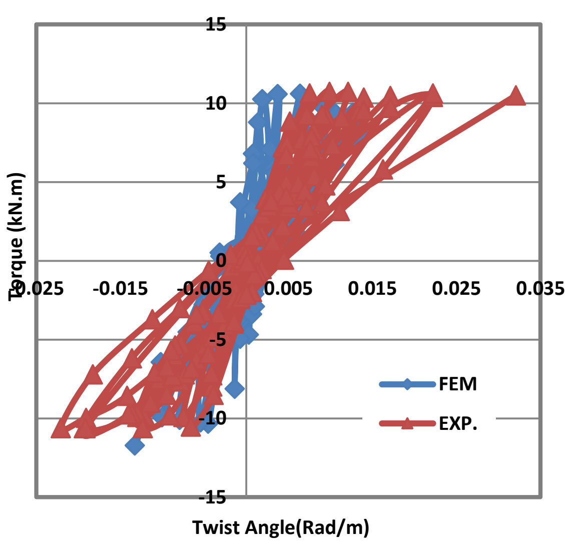

Figure 16. **Cyclic Torque-Twist Angle for Experimental and Numerical for Beam (C2).**

8. Conclusions

From the predicted results of nonlinear analysis of prestressed concrete beams under pure cyclic torsion, the predicted results of verification models used in finite element analysis using ANSYS software show a good agreement with the experimental results for torque - twist angle and ultimate torque at static and cyclic analysis.

Nomenclature

English System

Ec	Modulus of elasticity of concrete, MPa
f'c	The cylinder concrete compressive strength, MPa
fly	Yielding stress of longitudinal reinforcement, MPa
fp	Peak stress, MPa

fr	Modulus of rupture for concrete, MPa
fsy	Yielding stress of transverse reinforcement, MPa
ft	Splitting tensile strength for concrete, MPa
fy	Specified yield strength of steel reinforcement, MPa
S	Spacing of stirrups, mm
Tcr	Cracking torsional Strength, kN.m
Tu	Torque resistance at failure, kN.m
u, v, w	Displacement coordinates in x, y, and z Cartesian coordinates
x, y, z	Cartesian coordinates
$[\ddot{u}]$	Second derivatives of the displacement with respect to time
$[\dot{u}]$	First derivatives of the displacement with respect to time
[B]	Strain displacement matrix
[C]	Viscous damping matrix
[D]	Material stiffness matrix
[K]	Global stiffness matrix
[k]	Stiffness matrix for particular element
[M]	Mass matrix
	Greek System
{e}	The out of balance force vector
$\gamma_1, \gamma_2, \gamma_3$	Shear retention parameters
$Ni(\xi, \eta, \zeta)$	The element interpolation function corresponding to node i
v	Poisson's ratio
εc	Compressive principal strain for concrete
εc	Strain at cylinder strength fc'
εp	Peak strain
θ	Angle of inclination of the concrete stuts
{F}	Nodal loads
{p}	Vector of the nodal force equivalent to the internal stress level
{u}	Nodal displacements
{ε}	Element strains
{σ}	Elements stress

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